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# **Characteristic Impedance of Microstrip Lines**

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# Characteristic Impedance of Microstrip Lines

by

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## Abstract

The dyadic Green's function for a current embedded in a grounded dielectric slab is used to analyze microstrip lines at millimeter wave frequencies. The dyadic Green's function accounts accurately for fringing fields and dielectric cover over the microstrip line. Using Rumsey's reaction concept, an expression for the characteristic impedance is obtained. The numerical results are compared with results reported by others.

## Introduction

In recent years there has been a great deal of interest in analyzing microstrip lines at millimeter wave frequencies due to use of these lines in monolithic phased array systems.

Microstrip lines were initially analyzed using a quasi-static approach and later by a waveguide model to study dispersion characteristics [4]. However, these models do not take into account losses due to radiation and surface wave excitation. These losses can be significant at high frequencies and some attempts [1-3] have been made to account for these losses. In reference 1, the current distribution on the microstrip discontinuity is determined by solving the electric field integral equation. From a knowledge of the current distribution, characteristics such as impedance and guide wavelength can then be determined. The microstrip line embedded in a multilayer dielectric has also been analyzed using a generalized spectral domain Green's function [3].

In this paper, the dyadic Green's function for a current source embedded in a grounded dielectric slab is used to determine the field due to the microstrip line current. Using Rumsey's reaction concept [5], the characteristic impedance of the microstrip line embedded in a dielectric slab is then determined. The numerical results obtained using the present method are compared with earlier published data.

## Symbols

$d$	Total thickness of dielectric.
$\mathbf{E}$	Electric field vector.
$\mathbf{E}^i$	Incident electric field.
$f_0$	Frequency.
$f_x(y)$	Transverse distribution function.
$I_n$	Complex amplitude of $n^{\text{th}}$ pulse.
$I_0$	Total input current.
$\mathbf{J}(x,y)$	Surface electric current density vector.
$j$	$\sqrt{-1}$ .
$J_x(x,y)$	x-component of $\mathbf{J}(x,y)$ .
$J_x(k_x, k_y)$	Bidimensional Fourier transform of $J_x(x,y)$ .
$J_t, J_{xt}$	Surface current distribution on microstrip.
$J_u, J_{xu}$	Surface current distribution on microstrip induced by x-polarized plane wave.
$k$	Wave propagation constant in dielectric, $k_0 \sqrt{\epsilon_r}$ .
$k_0$	Wave propagation constant in free space.
$K_e$	Dominant mode propagation constant.
$k_x$	Fourier transform variable with respect to x.
$k_y$	Fourier transform variable with respect to y.
$N$	Number of pulses in y-direction.
$W$	Width of microstrip line.
$\mathbf{x}, \mathbf{y}, \mathbf{z}$	Unit vector along x-, y-, z-axis respectively.
$y_n$	$-W/2 + (2n-1)\Delta y/2$ .
$Z_0$	Characteristic impedance of microstrip line.
$z'$	Microstrip location from ground plane.
$\alpha$	Imaginary part of $K_e$ .
$\beta$	Real part of $K_e$ .
$\Delta y$	$W/N$ .
$\epsilon_r$	Relative permittivity of dielectric substrate.
$\lambda_0$	Wavelength in free space.
$\eta_0$	Characteristic impedance of free space.

## Theory

The geometry of a microstrip line embedded in a grounded dielectric slab is shown in figure 1 along with the notation to be used. In order to determine the characteristic impedance, consider a microstrip line being excited by a current source backed by a perfect electric conductor at the  $x=0$  plane, as shown in the current source looks into a semi-infinite transmission line, the characteristic impedance of the line would be the same as the input impedance seen by the source. Assuming the strip width,  $W$ , is much smaller than the operating wavelength, the  $y$ -component of the strip current can be neglected. The surface current,  $J(x,y)$ , on the  $z=z'$  plane may then be represented by

$$J(x,y) = x f(y) \exp(-jK_e |x|) \quad -\infty \leq x \leq \infty \quad (1)$$

where  $K_e = \beta + j\alpha$  is the dominant mode propagation constant along the strip. The range of  $x$  is extended to  $-\infty$  due to the image of the strip behind the conducting plane at  $x=0$  and the magnitude of  $x$  is used to indicate propagation away from the source at  $x=0$ . The transverse distribution,  $f(y)$ , in equation 1 may be assumed to be of the form

$$f(y) = \sum_{n=1}^N I_n P_n(y) \quad (2)$$

where

$$P_n(y) = 1/\Delta y \quad \text{for} \quad (y_n - \frac{\Delta y}{2}) \leq y \leq (y_n + \frac{\Delta y}{2})$$

$$= 0 \quad \text{otherwise.}$$

From the continuity condition, the distribution  $f(y)$

must satisfy

$$\int_{-W/2}^{W/2} f(y) dy = I_0 \quad (3)$$

The pulse amplitudes,  $I_n$ , in equation 2 are determined, as explained later, by using the integral equation method in conjunction with the method of moments.

If  $E(x, y, z)$  is the electric field due to the strip current,  $J(x, y)$ , then the input impedance seen by the source is obtained from

$$Z_{in} = Z_0 = - \frac{1}{I_0^2} \int_{-W/2}^{W/2} \int_0^\infty J(x, y) \bullet E(x, y, z') dx dy \quad (4)$$

This approach thus differs from the solution to the multilayer transmission line problem [3], where the characteristic impedance is obtained after dividing the average voltage by the total current on the strip. Furthermore, by taking into account the actual current distribution in the y-direction, it is expected that the present approach would give more accurate results. Using the dyadic Green's function for a current embedded in a grounded dielectric slab [6], the x-component of the electric field  $E(x, y, z)$  is obtained as

$$\begin{aligned} x \bullet E(x, y, z') = & \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Q(k_x, k_y) J_x(k_x, k_y) \\ & \bullet \exp(jk_x x + jk_y y) dk_x dk_y \quad (5) \end{aligned}$$

where,

$$Q(k_x, k_y) = (k^2 - k_x^2) \mathcal{G}_1(k_x, k_y) + j k_x^2 \mathcal{G}_2(k_x, k_y) \quad (6)$$

$$\mathcal{G}_1(k_x, k_y) = \left\{ \frac{\sin(k_I z')}{(k_I z')} \right\} \cdot \left\{ \frac{k_I \cos(k_I (d - z')) + j k_{II} \sin(k_I (d - z'))}{k_I \cos(k_I d) + j k_{II} \sin(k_I d)} \right\} \quad (7a)$$

$$\mathcal{G}_2(k_x, k_y) = \left\{ \frac{-k_I^2 \sin(k_I z')}{\epsilon_r k_{II} \cos(k_I d) + j k_I \sin(k_I d)} \right\} \cdot \left\{ \frac{\sin(k_I z')}{(k_I z')} \right\} \left\{ \frac{(\epsilon_r - 1)}{k_I \cos(k_I d) + j k_{II} \sin(k_I d)} \right\} \quad (7b)$$

$$k_I = \begin{cases} \sqrt{k^2 - k_x^2 - k_y^2} & (k_x^2 + k_y^2) \leq k^2 \\ -j \sqrt{k_x^2 + k_y^2 - k^2} & (k_x^2 + k_y^2) > k^2 \end{cases}$$

$$k_{II} = \begin{cases} \sqrt{k_0^2 - k_x^2 - k_y^2} & (k_x^2 + k_y^2) \leq k_0^2 \\ -j \sqrt{k_x^2 + k_y^2 - k_0^2} & (k_x^2 + k_y^2) > k_0^2 \end{cases}$$

In equation 5,  $\eta_0$  is the free space characteristic impedance,  $\mathcal{G}_1$  and  $\mathcal{G}_2$  are derived in reference 6, and  $J_x(k_x, k_y)$  is given by

$$J_x(k_x, k_y) = \int_{W/2 - \infty}^{W/2 + \infty} \int_{-\infty}^{\infty} f(y) \exp(-jK_e |x| - jk_x x - jk_y y) dx dy \quad (8)$$

From equations 2 and 8, and assuming  $\alpha$  to be a finite

negative value, the integral in equation 8 may be evaluated in closed form as

$$J_x(k_x, k_y) = \sum_{n=1}^N jI_n \left[ \frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)} \right] \cdot \left[ \frac{1}{k_x - K_e} - \frac{1}{k_x + K_e} \right] \exp(-jk_y y_n) \quad (9)$$

Substituting equation 5 into equation 4 and after some mathematical manipulations, the characteristic impedance is obtained as

$$Z_0 = \frac{j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^N \frac{jI_n}{I_0^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[ \frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)} \right] \cdot \left[ \frac{1}{k_x - K_e} - \frac{1}{k_x + K_e} \right] Q(k_x, k_y) \cdot J_x(-k_x, -k_y) \exp(-jk_y y_n) \right\} dk_x dk_y \quad (10)$$

where

$$J_x(-k_x, -k_y) = \int_{-W/2}^{W/2} \int_0^{\infty} f(y) \exp(-jK_e x + jk_x x + jk_y y) dx dy = \sum_{m=1}^N jI_m \left[ \frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)} \right] \left[ \frac{\exp(jk_y y_m)}{k_x - K_e} \right] \quad (11)$$

Substituting equation 11 into equation 10 gives

$$\begin{aligned}
Z_0 = & \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^N \sum_{m=1}^N \frac{I_n I_m}{I_0^2} \\
& \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[ \frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)} \right]^2 \left[ \frac{1}{(k_x - K_e)^2} - \frac{1}{k_x^2 - K_e^2} \right] \right. \\
& \left. \cdot Q(k_x, k_y) \exp(jk_y(y_m - y_n)) \right\} dk_x dk_y \quad (12)
\end{aligned}$$

Making a change of variables such that:

$$\zeta = \frac{k_x}{k_0} ; \quad \xi = \frac{k_y}{k_0} ; \quad \nu = \frac{K_e}{k_0} ,$$

equation 12 is rewritten as

$$\begin{aligned}
Z_0 = & \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^N \sum_{m=1}^N \frac{I_n I_m}{I_0^2} \\
& \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[ \frac{\sin(\xi k_0 \Delta y/2)}{(\xi k_0 \Delta y/2)} \right]^2 \left[ \frac{1}{(\zeta - \nu)^2} - \frac{1}{\zeta^2 - \nu^2} \right] \right. \\
& \left. \cdot Q(\zeta, \xi) \exp(j\xi k_0(y_m - y_n)) \right\} d\zeta d\xi \quad (13)
\end{aligned}$$

where,  $Q(\zeta, \xi) \equiv Q(k_0 \zeta, k_0 \xi)$ .

Note that the integrand in equation 13 has poles at  $\zeta = \nu$  of order two and simple poles at  $\zeta = \pm \nu$ . The integration with respect to  $\zeta$  can be evaluated using the residue theorem. In order to evaluate the integration with respect to  $\zeta$ , consider the following contour integration,

$$\int_C \left\{ \left( \frac{1}{\gamma - \nu} \right)^2 - \left( \frac{1}{\gamma^2 - \nu^2} \right) \right\} Q(\gamma, \xi) d\gamma \quad (14)$$

where  $\zeta$  is the real part of  $\gamma$ , and  $C$  is the closed contour to be selected. The function  $Q(\gamma, \xi)$  has branch cuts at

$$\gamma = \pm \sqrt{1 - \xi^2}$$

and poles corresponding to the roots of the equations:

$$\begin{aligned} k_I \cos(k_I d) + j k_{II} \sin(k_I d) &= 0 \\ \epsilon_r k_{II} \cos(k_I d) + j k_I \sin(k_I d) &= 0 \end{aligned}$$

These poles correspond to the surface wave modes that would be launched by the semi-infinite microstrip line. These surface wave poles and the branch cuts may be plotted in the complex  $\gamma$ -plane as shown in figure 3. The contour  $C$  should be selected such that the propagation of the surface waves in the  $x > 0$  direction is assured. For this condition, the surface wave poles on the positive real axis must be included in the contour  $C$ . The required contour  $C$  is shown in figure 3. The integration in equation 14 can be written as

$$\int_C = \int_{C_1} + \int_{C_2} + \int_{C_3} = -2\pi j \{ R_1 + R_2 + R_3 \} \quad (15)$$

where,

$$\begin{aligned} R_1 &= \sum \text{Residues of } Q \text{ at surface wave poles} \\ R_2 &= \text{Residue of } Q/(\gamma - \nu)^2 \text{ at } \gamma = \nu \\ R_3 &= \text{Residue of } Q/(\gamma^2 - \nu^2) \text{ at } \gamma = \nu. \end{aligned}$$

In equation 15, the integration along  $C_2$  is zero because of the radiation condition, the integration along the branch

cut,  $C_3$ , represents a radiation field, and the integration along  $C_1$  is the required integration to be evaluated with respect to  $\zeta$ . Since the radiation field due to the microstrip line would be very small, the integration along  $C_3$  will be neglected. For the present case, the contribution due to surface wave modes will also be neglected. The integration with respect to  $\zeta$  becomes

$$\begin{aligned} & \int_{-\infty}^{\infty} \left\{ \left( \frac{1}{\zeta - \nu} \right)^2 - \left( \frac{1}{\zeta^2 - \nu^2} \right) \right\} Q(\zeta, \xi) d\zeta \\ &= -2\pi j \left\{ \left. \frac{\partial Q(\zeta, \xi)}{\partial \zeta} \right|_{\zeta=\nu} - \frac{Q(\zeta, \xi)}{2\zeta} \right|_{\zeta=\nu} \right\} \quad (16) \end{aligned}$$

Substituting equation 16 into equation 13, the expression for  $Z_0$  reduces to

$$\begin{aligned} Z_0 = & \frac{-\eta_0 k_0 z'}{2\pi k^2} \sum_{n=1}^N \sum_{m=1}^N \frac{I_n I_m}{I_0^2} \int_0^{\infty} \left\{ \left[ \frac{\sin(k_0 \xi \Delta y / 2)}{(k_0 \xi \Delta y / 2)} \right]^2 \right. \\ & \cdot \left. \left( \frac{2\partial Q(\zeta, \xi)}{\partial \zeta} - \frac{Q(\zeta, \xi)}{\zeta} \right) \right|_{\zeta=\nu} \cos(\xi(m-n)k_0 \Delta y) \Big\} d\xi \quad (17) \end{aligned}$$

From the characteristic equation (27), it can be seen that the term

$$\begin{aligned} & \sum_{n=1}^N \sum_{m=1}^N \frac{I_n I_m}{I_0^2} \int_0^{\infty} \left\{ \left. \frac{Q(\zeta, \xi)}{\zeta} \right|_{\zeta=\nu} \right. \\ & \cdot \left. \left[ \frac{\sin(k_0 \xi \Delta y / 2)}{(k_0 \xi \Delta y / 2)} \right]^2 \cos(\xi(m-n)k_0 \Delta y) \right\} d\xi \end{aligned}$$

in equation 17 is always zero.

For a uniform current distribution in the y-direction ( $I_n = I_0$ ), the expression in equation 17 becomes

$$Z_0 = \frac{-\eta_0 k_0 z'}{\pi k^2} \int_0^\infty \left\{ \left. \frac{\partial Q(\zeta, \xi)}{\partial \zeta} \right|_{\zeta=\nu} \left[ \frac{\sin(k_0 \xi \Delta y / 2)}{(k_0 \xi \Delta y / 2)} \right]^2 \right\} d\xi \quad (18)$$

For a nonuniform current distribution of

$$f(y) = \frac{2I_0/(\pi W)}{\sqrt{1 - (2y/W)^2}} \quad (19)$$

the characteristic impedance of the transmission line becomes

$$Z_0 = \frac{-\eta_0 k_0 z'}{\pi k^2} \int_0^\infty \left\{ \left. \frac{\partial Q(\zeta, \xi)}{\partial \zeta} \right|_{\zeta=\nu} J_0^2(\xi k_0 W / 2) \right\} d\xi \quad (20)$$

where  $J_0(\cdot)$  denotes the Bessel function of the first kind of zero order.

### Evaluation of $K_e$ and $I_n$

In order to determine  $K_e$ , consider the infinite microstrip line as shown in figure 1. The surface current distribution on the strip may be assumed to be of the form

$$J_t(x,y) = x J_{xt}(x,y) = x f(y) \exp(-jK_e x) \quad -\infty \leq x \leq \infty \quad (21)$$

where  $f(y)$  is as given in equation 2.

Using equation 8, the Fourier transform of  $J_t(x,y)$  is obtained as

$$\begin{aligned} J_t(k_x, k_y) &= x J_{xt}(k_x, k_y) \\ &= x \sum_{n=1}^N I_n 2\pi \left[ \frac{\sin(k_y \Delta y / 2)}{(k_y \Delta y / 2)} \right] \\ &\quad \cdot \delta(k_x + K_e) \exp(-jk_y y_n) \end{aligned} \quad (22)$$

where  $\delta(\cdot)$  is a delta function and is the result of assuming a traveling wave current in the x-direction on the strip as denoted by the exponential factor in equation 21.

The x-component of the electric field radiated by  $J_{xt}$  is obtained by replacing  $J_x(k_x, k_y)$  with  $J_{xt}(k_x, k_y)$  in equation 5. Then by equating the x-component of the electric field to zero on the surface of the strip, since the total field must be zero and there is no incident field, an electric field integral equation is obtained

$$\begin{aligned}
0 = & \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^N I_n 2\pi \\
& \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[ \frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)} \right] Q(k_x, k_y) \delta(k_x + K_e) \right. \\
& \cdot \exp(-jk_y y_n) \exp(jk_x x + jk_y y) \left. \right\} dk_x dk_y \quad (23)
\end{aligned}$$

By setting  $x=0$  in equation 23 and selecting  $P_m(y)$  as a testing function and multiplying both sides of equation 23 by  $P_m(y)$  yields

$$\begin{aligned}
0 = & \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^N I_n 2\pi \\
& \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[ \frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)} \right] Q(k_x, k_y) \delta(k_x + K_e) \right. \\
& \cdot P_m(y) \exp(-jk_y y_n) \exp(jk_y y) \left. \right\} dk_x dk_y \quad (24)
\end{aligned}$$

Integrating both sides of equation 24 with respect to  $y$  and rearranging gives

$$\begin{aligned}
0 = & \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^N I_n 2\pi \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[ \frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)} \right] \right. \\
& \cdot Q(k_x, k_y) \delta(k_x + K_e) \exp(-jk_y y_n) \\
& \cdot \left[ \int_{Y_m - \Delta y/2}^{Y_m + \Delta y/2} P_m(y) \exp(jk_y y) dy \right] \left. \right\} dk_x dk_y \quad (25)
\end{aligned}$$

Using the property,

$$\int_{-\infty}^{\infty} f(x) \delta(x+x') dx = f(-x')$$

of a delta function, equation 25 can be written in the form

$$0 = \frac{-j\eta_0 k_0 z'}{2\pi k^2} \sum_{n=1}^N I_n \int_{-\infty}^{\infty} \left\{ Q(k_x, k_y) \Big|_{k_x = -K_e} \cdot \left[ \frac{\sin(k_y \Delta y / 2)}{(k_y \Delta y / 2)} \right]^2 \exp(-jk_y (y_n - y_m)) \right\} dk_y \quad (26)$$

Since  $Q(k_x, k_y) = Q(k_x, -k_y)$ , equation 26 may be rewritten as

$$0 = \frac{-j\eta_0 z'}{\pi \epsilon_r} \sum_{n=1}^N I_n \int_0^{\infty} \left\{ Q(\zeta, \xi) \Big|_{\zeta = -\nu} \cdot \left[ \frac{\sin(\xi k_0 \Delta y / 2)}{(\xi k_0 \Delta y / 2)} \right]^2 \cos(\xi k_0 \Delta y (m-n)) \right\} d\xi \quad (27)$$

where  $\xi = k_y / k_0$ ,  $\zeta = k_x / k_0$ ,  $\nu = K_e / k_0$ , and  $Q(\zeta, \xi) = Q(k_0 \zeta, k_0 \xi)$ .

By selecting  $m=1, 2, 3 \dots N$ , equation 27 may be written in matrix form as

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} & \dots & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} & \dots & Z_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & Z_{N3} & \dots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix} \quad (28)$$

where the components of the square matrix are given by

$$Z_{mn} = \int_0^{\infty} \left\{ Q(\zeta, \xi) \Big|_{\zeta=-\nu} \left[ \frac{\sin(\xi k_0 \Delta y / 2)}{(\xi k_0 \Delta y / 2)} \right]^2 \cdot \cos(\xi k_0 \Delta y (m-n)) \right\} d\xi \quad (29)$$

For a nontrivial solution of equation 28, the determinant of the square matrix in equation 28 must be set to zero, therefore,

$$\begin{vmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} & \cdots & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} & \cdots & Z_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & Z_{N3} & \cdots & Z_{NN} \end{vmatrix} = 0 \quad (30)$$

Equation 30 is solved for  $K_e$  in a numerical procedure by selecting  $N$  and then starting with an estimate of  $K_e \approx k$  and iterating  $K_e$  until equation 30 is satisfied to the desired accuracy.

In order to determine the current distribution in the transverse direction, it is assumed that the infinite microstrip line shown in figure 1 is excited by an x-polarized plane wave. The surface current induced on the strip may be assumed to be of the form

$$J(x, y) = \hat{x} J_{xu}(x, y) = \sum_{n=1}^N I_n P_n(y) \hat{x} \quad (31)$$

If  $E_{xu}(x, y, z)$  is the x-component of the scattered electric field due to  $J_{xu}(x, y)$ , and  $E_x^i(x, y, z)$  is the incident electric field, the boundary condition of zero tangential electric field on the conducting strip gives

$$-E_x^1(x, y, z') = E_{xu}(x, y, z') \quad (32)$$

$E_{xu}(x, y, z')$  can be obtained from equations 5, 6 and 8. Equation 32 then becomes

$$-E_x^1(x, y, z') = \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^N I_n \int_{-\infty}^{\infty} \left\{ Q(k_x, k_y) \right|_{k_x=0} \cdot \left[ \frac{\sin(k_y \Delta y / 2)}{(k_y \Delta y / 2)} \right] \exp(-jk_y y_n) \right\} dk_y \quad (33)$$

Equation 33 is an integral equation with  $I_n$  as the unknown complex values to be determined. Equation 33 can be solved by selecting  $P_m(y)$  as a testing function and applying the method of moments to yield

$$\sum_{n=1}^N I_n Z_{mn} = -\Delta y E_0^1(z') \quad \text{for } m=1, 2, 3, \dots, N \quad (34)$$

where  $E_0^1(z')$  is the x-polarized plane-wave electric field incident on the strip from the z-direction (normal incidence) and  $Z_{mn}$  is given by

$$Z_{mn} = \frac{-j\eta_0 k_0^2 z' \Delta y}{\pi \epsilon_r} \int_0^{\infty} \left\{ Q(\zeta, \xi) \right|_{\zeta=0} \cdot \left[ \frac{\sin(\xi k_0 \Delta y / 2)}{(\xi k_0 \Delta y / 2)} \right] \cos(\xi(m-n)k_0 \Delta y) \right\} d\xi \quad (35)$$

Equation 34 can be written in matrix notation as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} & \cdots & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} & \cdots & Z_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & Z_{N3} & \cdots & Z_{NN} \end{bmatrix} = \begin{bmatrix} -\Delta y & E_0^i \\ -\Delta y & E_0^i \\ -\Delta y & E_0^i \\ \vdots & \vdots \\ -\Delta y & E_0^i \end{bmatrix} \quad (36)$$

and the complex values of  $I_n$  are obtained by matrix inversion.

## Numerical Results

In order to obtain numerical results, the semi-infinite integrals appearing in equations 17, 18, 20, 29 and 35 are evaluated using Gauss Quadrature integration techniques and the upper limit on the integration is truncated to a large finite value. From numerical results, it was found that the upper limit on the integration depended upon frequency and could vary from  $t \approx 200$  (for high frequencies) to  $t \approx 1000$  (for frequencies below 5 GHz). Because of the complexity of the expression for  $Q(k_x, k_y)$ , the derivative,  $\partial Q / \partial k_x$  or  $\partial Q / \partial \zeta$ , is evaluated numerically.

In order to obtain meaningful numerical results, it is desirable to apply convergence tests to  $I_n$ ,  $K_e$  and  $Z_0$ . The matrix equation is solved for  $I_n$  for various values of  $N$  and plotted in figure 4. It is observed that a stable solution for  $I_n$  is obtained when  $N \geq 25$ . Figure 5 shows the variation of  $K_e$  and  $Z_0$  as a function of  $N$ . It is noted that  $N \geq 25$  also gives converged values for these quantities.

To verify the present formulation, the characteristic impedance and the guide wavelength of a microstrip line with  $W/z' = 1.0$ ,  $d = z' = -.04$  cm, and  $\epsilon_r = 10$ , is computed using equations 13 and 22 for  $N = 25$ . These results are presented in figure 6 as a function of frequency along with results reported earlier [5]. Using the present method,  $Z_0$  and  $K_e$  are also computed for uniform current distribution and nonuniform (see eq.14) current distributions and presented in figure 6. The results obtained by the present method with uniform distribution are in excellent agreement with the results reported earlier [4]. However,  $Z_0$  and  $K_e$  calculated by taking into account the actual current distribution (as obtained from equation 36) is believed to give more accurate results than the results obtained with

uniform current distribution. It is also observed from curves C & D of figure 6, that the assumed nonuniform distribution is close to the actual calculated current distribution and may be used instead of the pulse distribution in order to save computation time. To compare the present method with the approach in reference 3, the characteristic impedance of a microstrip line with  $d=z'$  and  $d=4z'$ ,  $\epsilon_r=2.53$ ,  $f_0=3\text{GHz}$  is computed as a function of strip width,  $W/z'$ , and presented in figures 7 and 8. It is noted that there is a small discrepancy between the two results in figure 8. This may be attributed to the different type of current distribution assumed in the two cases. With an assumption of uniform current distribution in the y-direction, the results obtained by the present method are also presented in figure 7. There is good agreement between the results obtained by the two methods. Figure 8 shows the characteristic impedance of a microstrip line covered with a dielectric slab.

The results obtained by the present method are also compared in figure 9 with data taken from reference 1. For frequencies greater than 42 GHz, the dielectric thickness becomes greater than a quarter-wavelength and the current distribution in the y-direction may not be of the assumed form. Furthermore, for thicknesses greater than a quarter-wavelength, the microstrip line would act as a radiator rather than a transmission line.

It is interesting to study the effect of distribution in the y-direction on the guide wavelength and the characteristic impedance of a microstrip line. It is expected that for a narrow strip, the distribution may be closer to the assumed nonuniform function (equation 19); whereas, for a wider strip, the distribution is expected to be closer to uniform, except near the strip edges. To

verify this expectation, the guide wavelength and the characteristic impedance of a microstrip line with  $d=z'=-.05$  wavelengths,  $\epsilon_r=10$ , and the strip width in the range of 0.025 to 0.330 wavelengths were computed and presented in figure 10. The guide wavelength computed with the assumed nonuniform distribution and pulse distribution are almost the same for a narrow strip ( $W/\lambda_0 < 0.05$ ). For wider strips ( $W/\lambda_0 > 0.27$ ) the guide wavelength computed using the uniform distribution and the pulse distribution approach the same values. From figure 10, it is observed that the characteristic impedance does not significantly depend upon the transverse distribution of the current.

For microwave integrated circuit design, alumina substrate with dielectric constants in the range of 10 to 12.8 are often used. In order to design a microstrip line on these substrates, an accurate knowledge of the guide wavelength and the characteristic impedance is essential. In figures 11 and 12, the guide wavelength and the characteristic impedance are presented as a function of frequency for various design parameters. Calculations for other parameters may be obtained with the aid of the computer code given in the appendix.

## Conclusion

Using the dyadic Green's function and Rumsey's reaction concept, an expression for the characteristic impedance of a microstrip line embedded in a grounded dielectric slab has been derived. The characteristic impedance determined by the present method is found to be in excellent agreement with earlier published results. Numerical results for a microstrip line on alumina substrate is presented at millimeter wave frequencies. A computer code listing is included for calculating the characteristic impedance and propagation constant for microstrip lines.

## References

1. P. B. Katehi and N. G. Alexopoulos: "Frequency-Dependent Characteristics of Microstrip Discontinuities in Millimeter Wave Integrated Circuits", IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-33, No. 10, pp. 1029-1035, October 1985.
2. R. W. Jackson and D. M. Pozar: "Full Wave Analysis of Microstrip Open End and Gap Discontinuities", IEEE Trans. on Microwave Theory and Techniques, Vol MTT-33, No. 10, pp. 1036-1042, October 1985.
3. N. K. Das and D. M. Pozar: "A Generalized Spectral Domain Green's Function for Multilayer Dielectric Substrates with Application to Multilayer Transmission Lines", IEEE Trans. on Microwave Theory and Techniques, Vol MTT-35, No. 3, pp. 326-335, March 1987.
4. K. C. Gupta, R. Garg and I. J. Bahl: *Microstrip Lines and Slotlines*, Artech House, Dedham, MA, 1979.
5. R. F. Harrington: *Time Harmonic Electromagnetic Fields*, McGraw-Hill Book Company, NY, 1961.
6. M. D. Deshpande and M. C. Bailey: "Integral Equation Formulation of Microstrip Antennas", IEEE Trans. on Antennas and Propagation, Vol. APS-30, No. 4, July 1982.

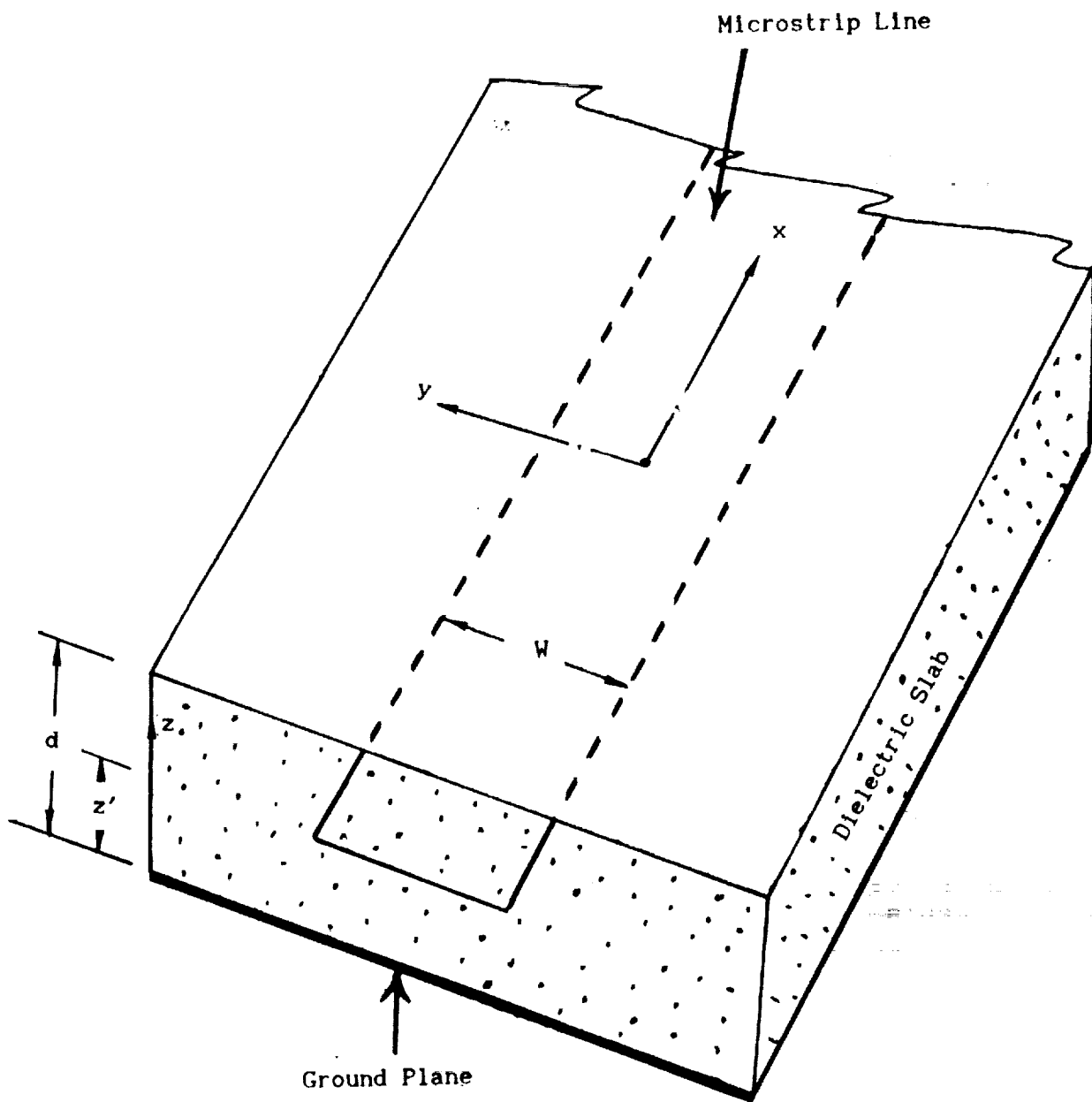


Figure 1. Geometry of microstrip line.

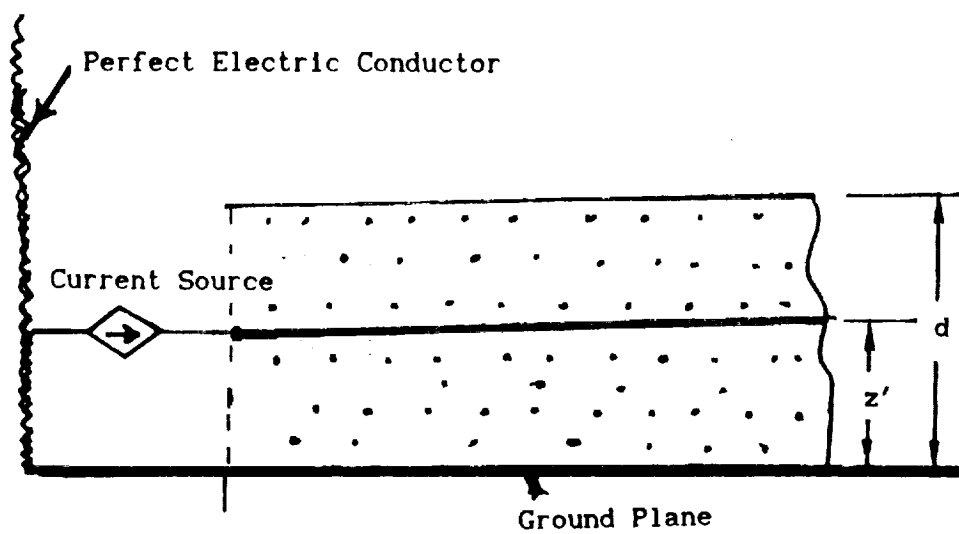


Figure 2. Current source exciting microstrip line.

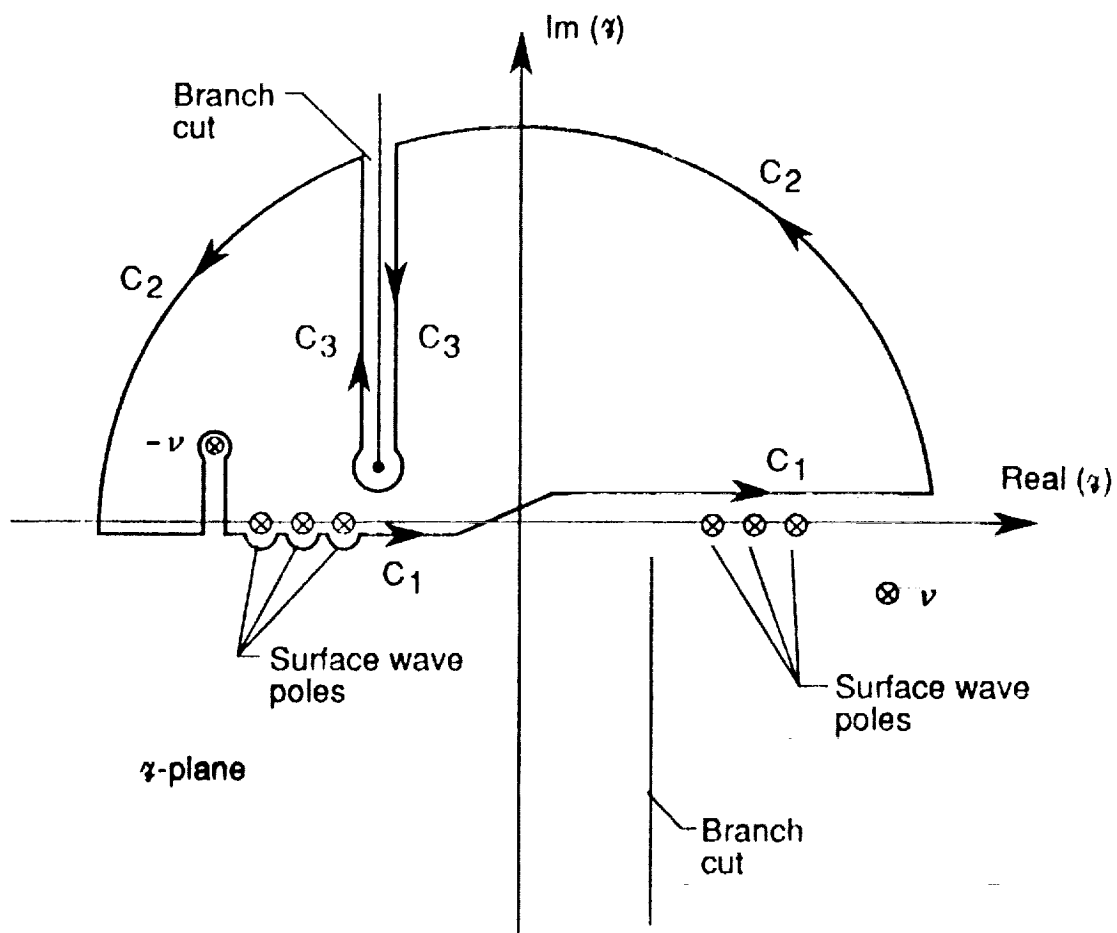


Figure 3. Complex  $\gamma$ -plane with surface wave poles and branch cuts.

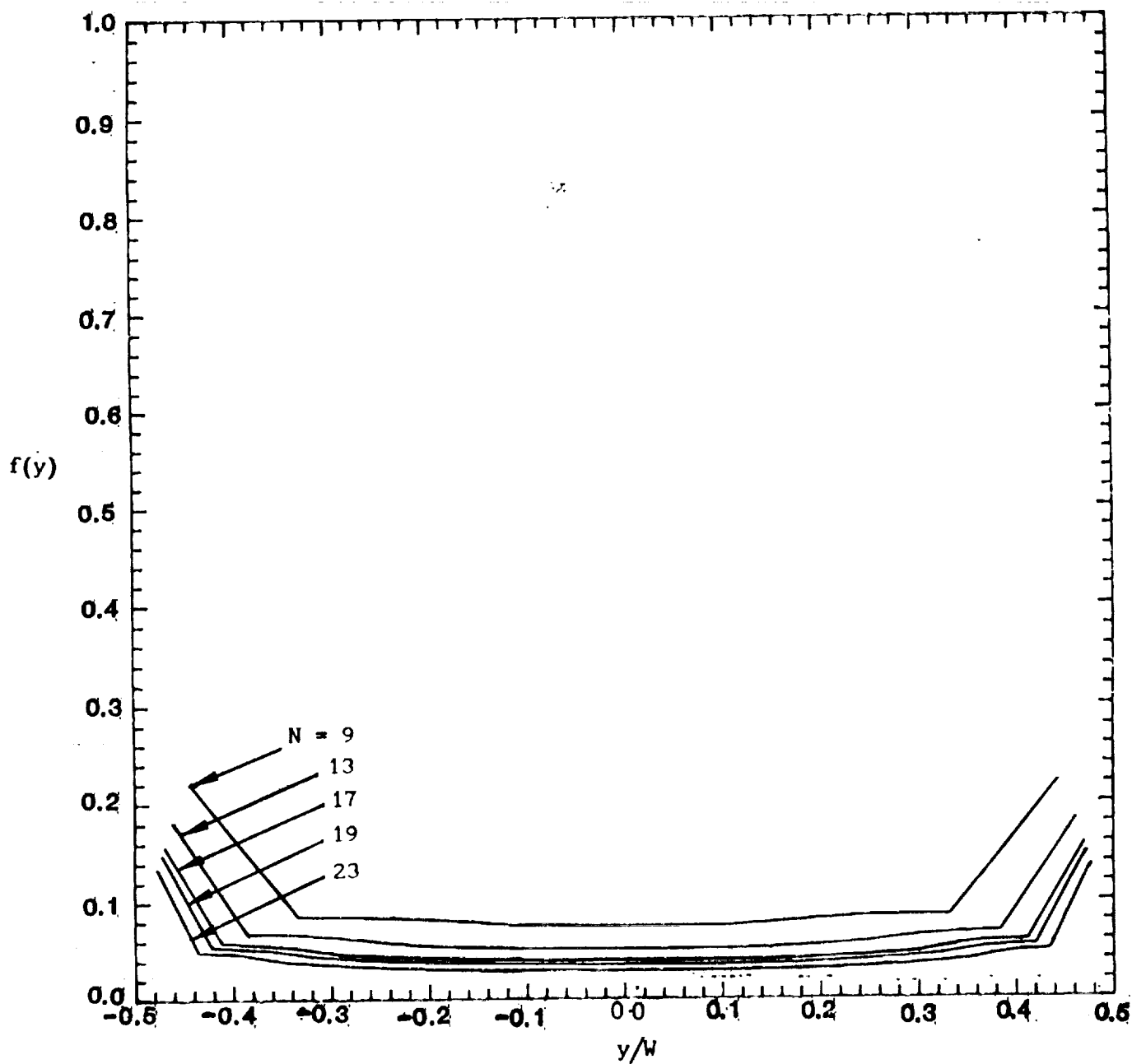


Figure 4. Current distribution in the transverse direction versus number of pulses. ( $\epsilon_r = 10$ ,  $W = z'$ ,  $z' = d = 0.08$  cm,  $f = 11.936$  GHz).

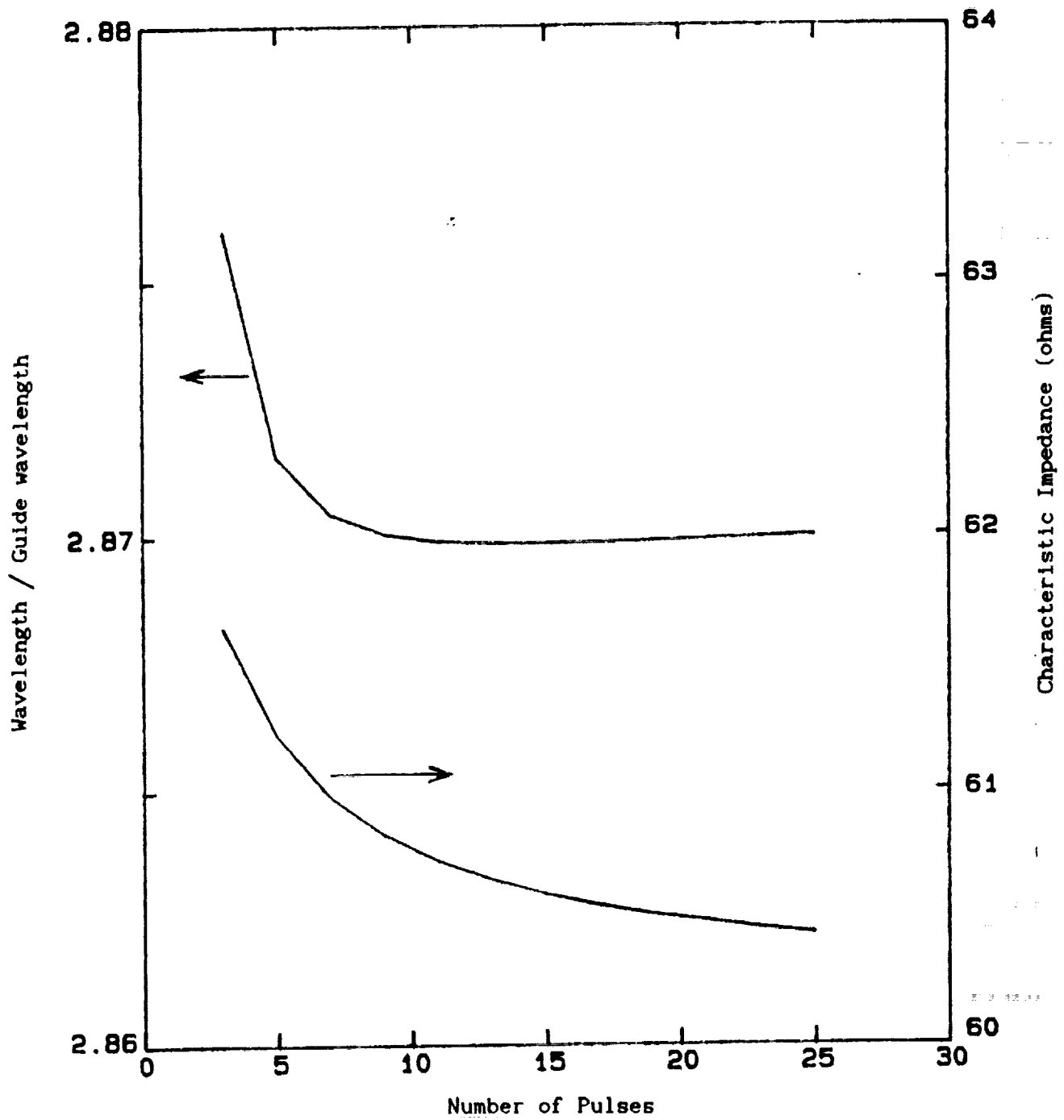


Figure 5. Variation of characteristic impedance and normalized guide wavelength of microstrip line as a function of number of transverse pulses. ( $\epsilon_r = 10$ ,  $W = z'$ ,  $z' = d = 0.04$  cm,  $f = 60$  GHz).

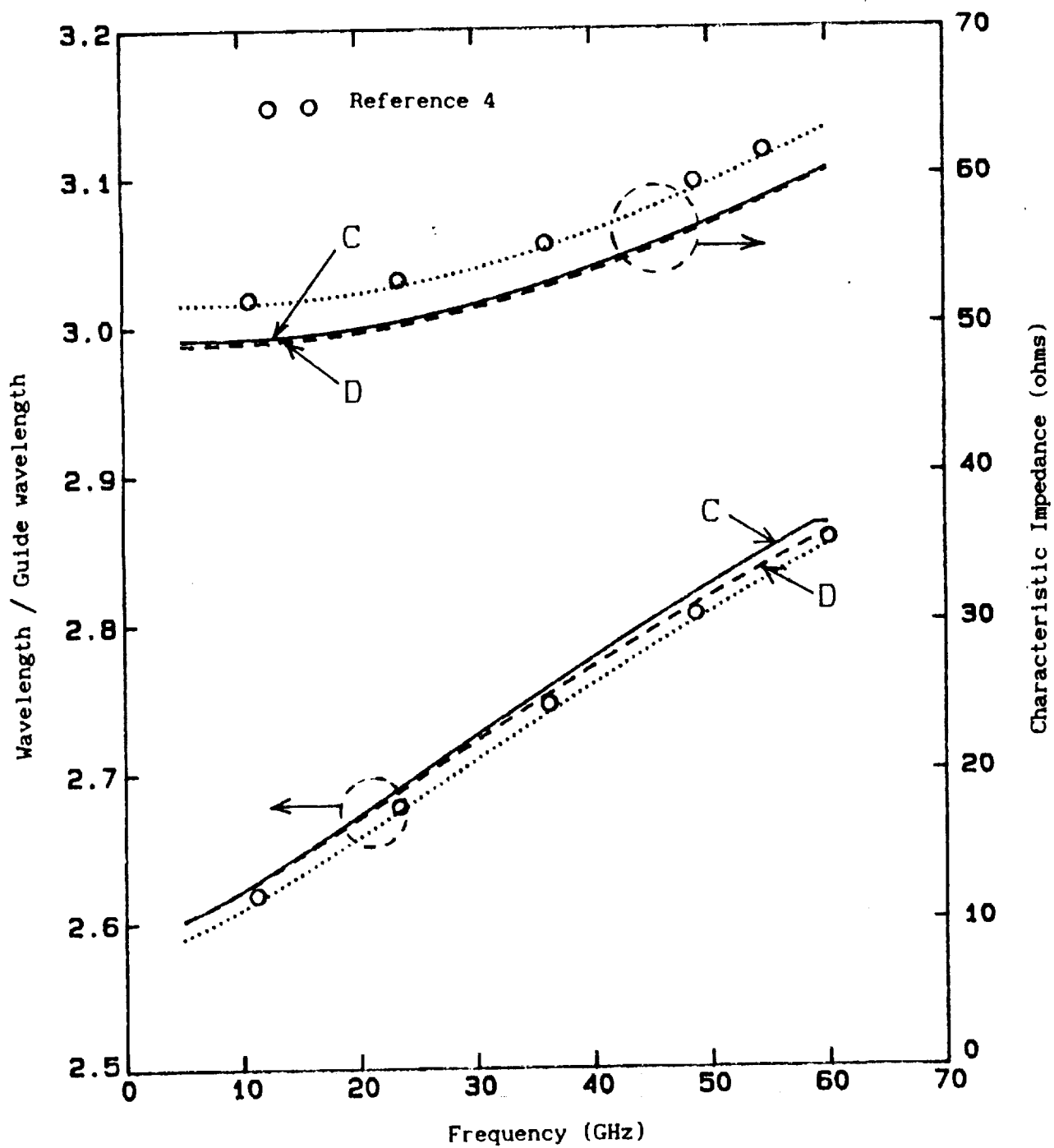


Figure 6. Characteristic impedance and normalized guide wavelength of microstrip line as a function of frequency. (..... uniform, - - - - nonuniform, ——— N=25 pulses).

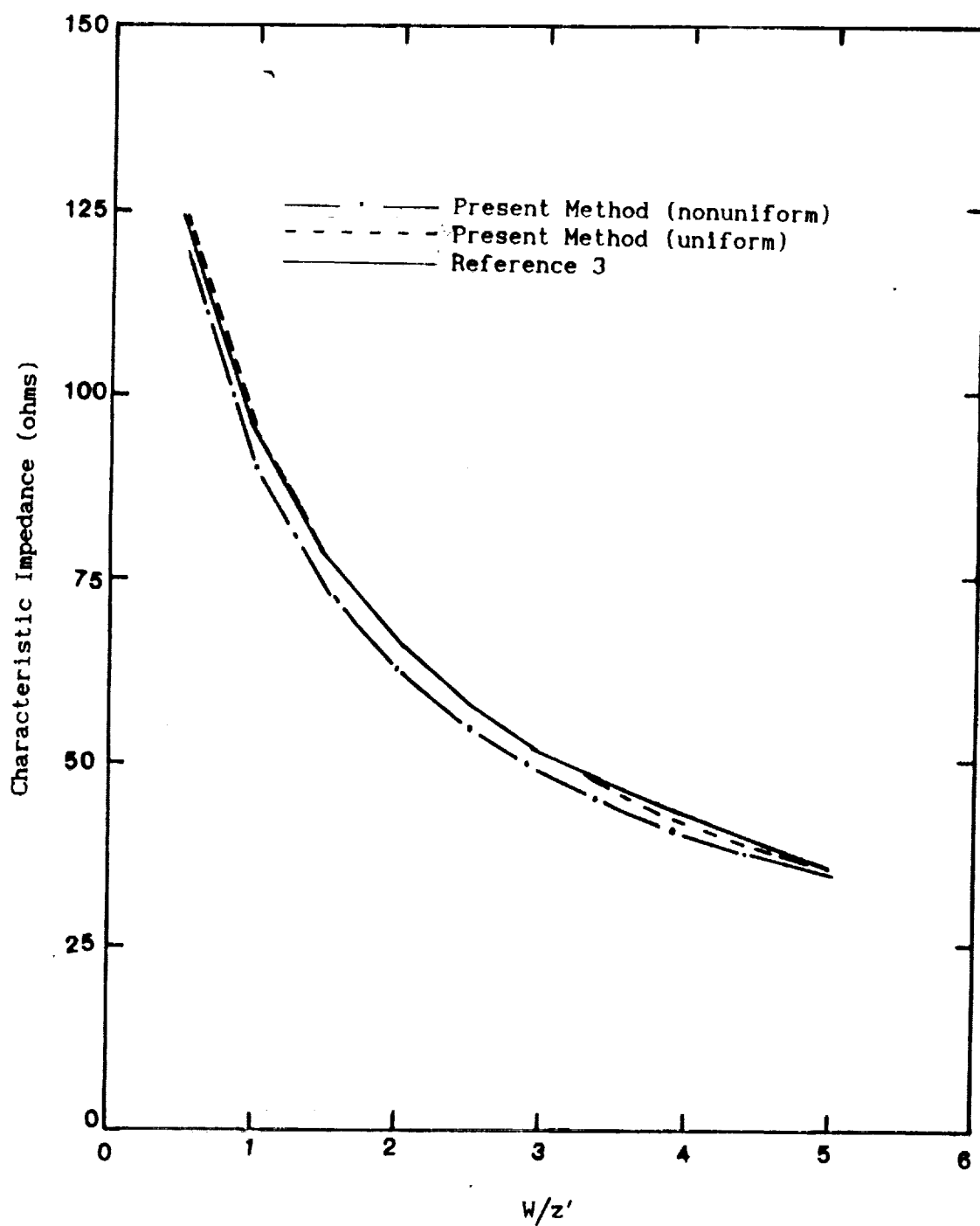


Figure 7. Characteristic Impedance of covered microstrip line.  
 ( $d = 0.64$  cm,  $z' = 0.16$  cm,  $\epsilon_r = 2.53$ ,  $f = 3$  GHz).

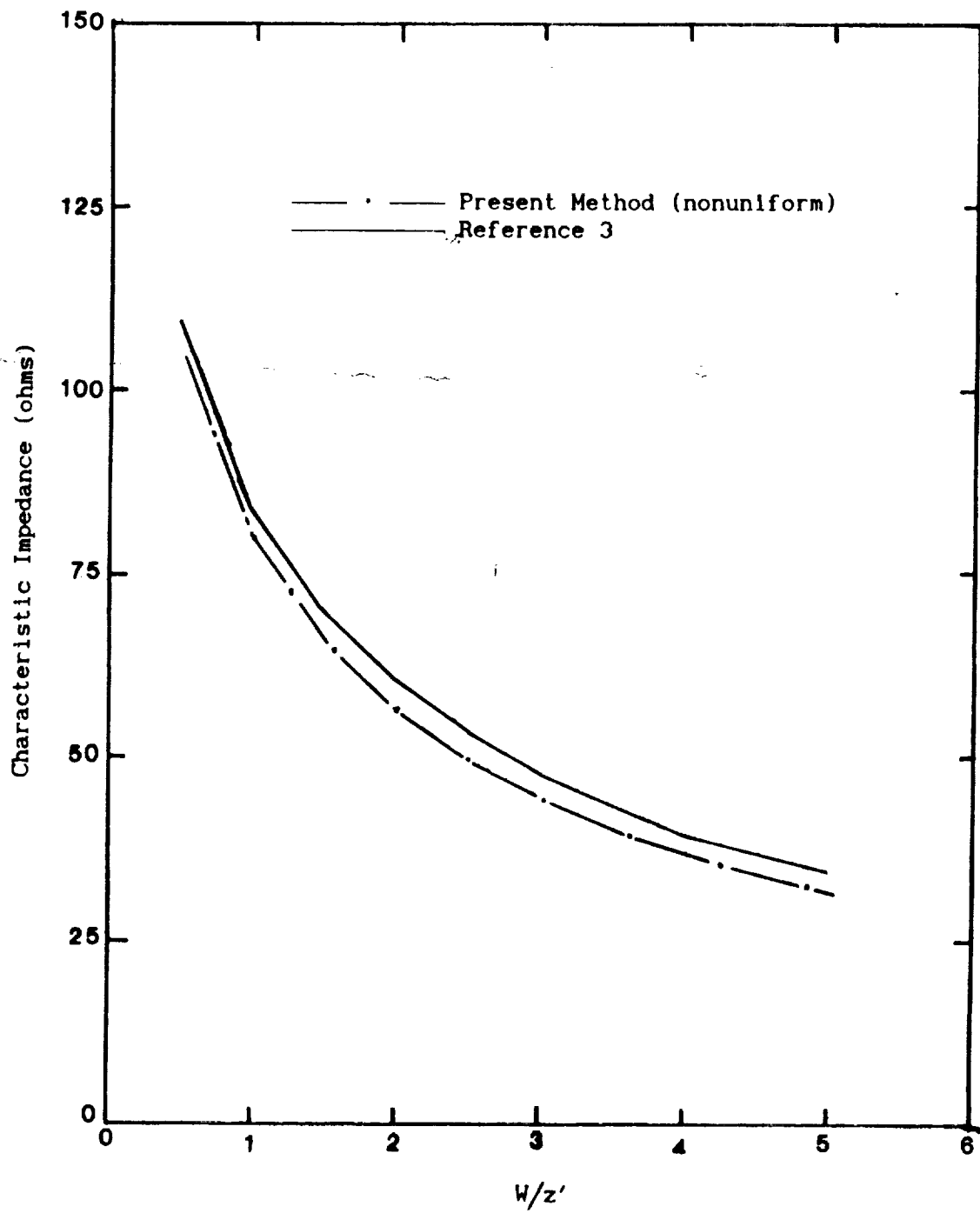


Figure 8. Characteristic impedance of microstrip line.  
( $z' = d = 0.16$  cm,  $\epsilon_r = 2.53$ ,  $f = 3$  GHz).

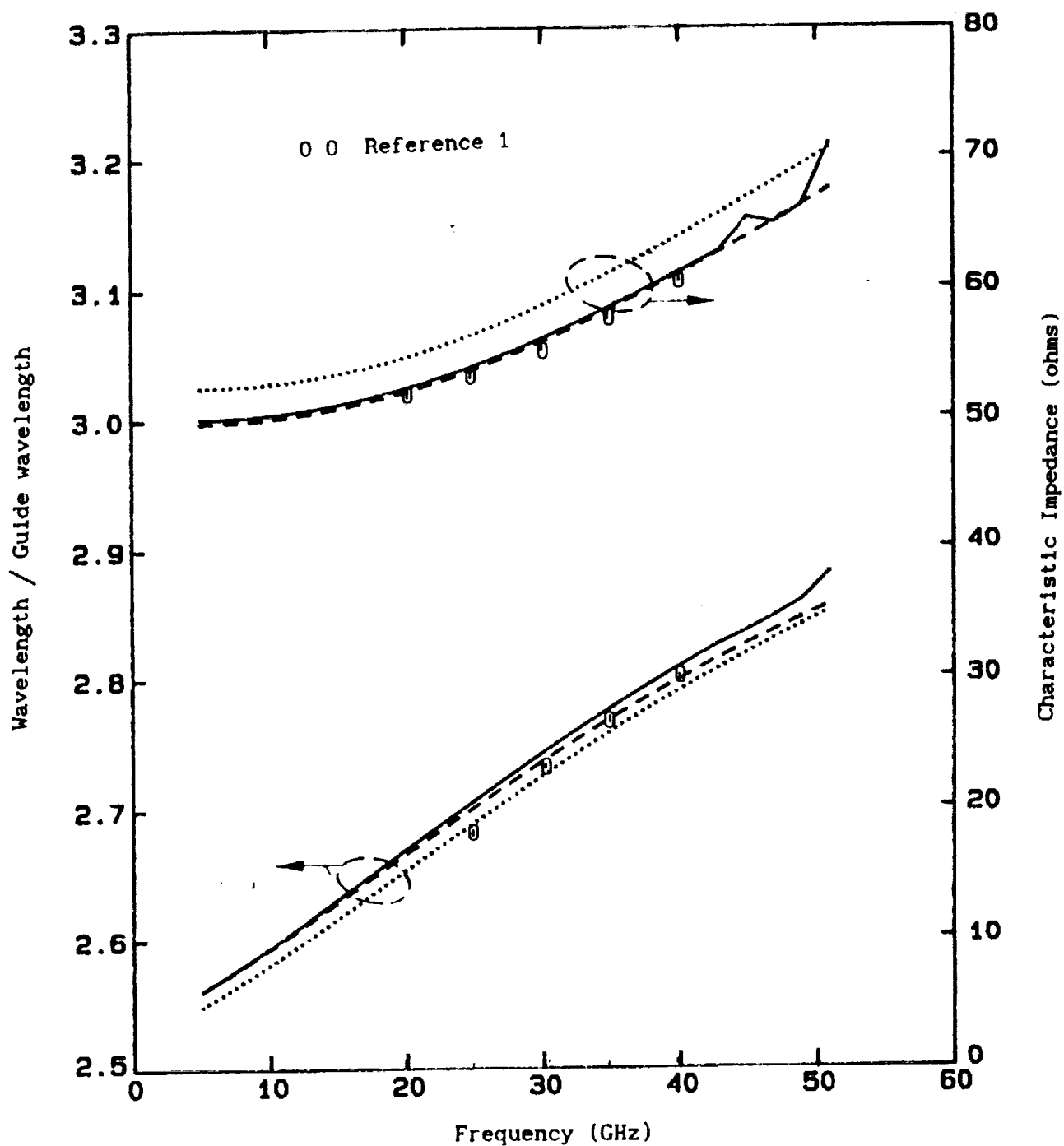


Figure 9. Characteristic impedance and normalized guide wavelength of microstrip line as a function of frequency.  
 $(\epsilon_r = 9.6, W = z', z' = d = 0.6 \text{ cm})$   
 (..... uniform, - - - - nonuniform, ——— N=25 pulses).

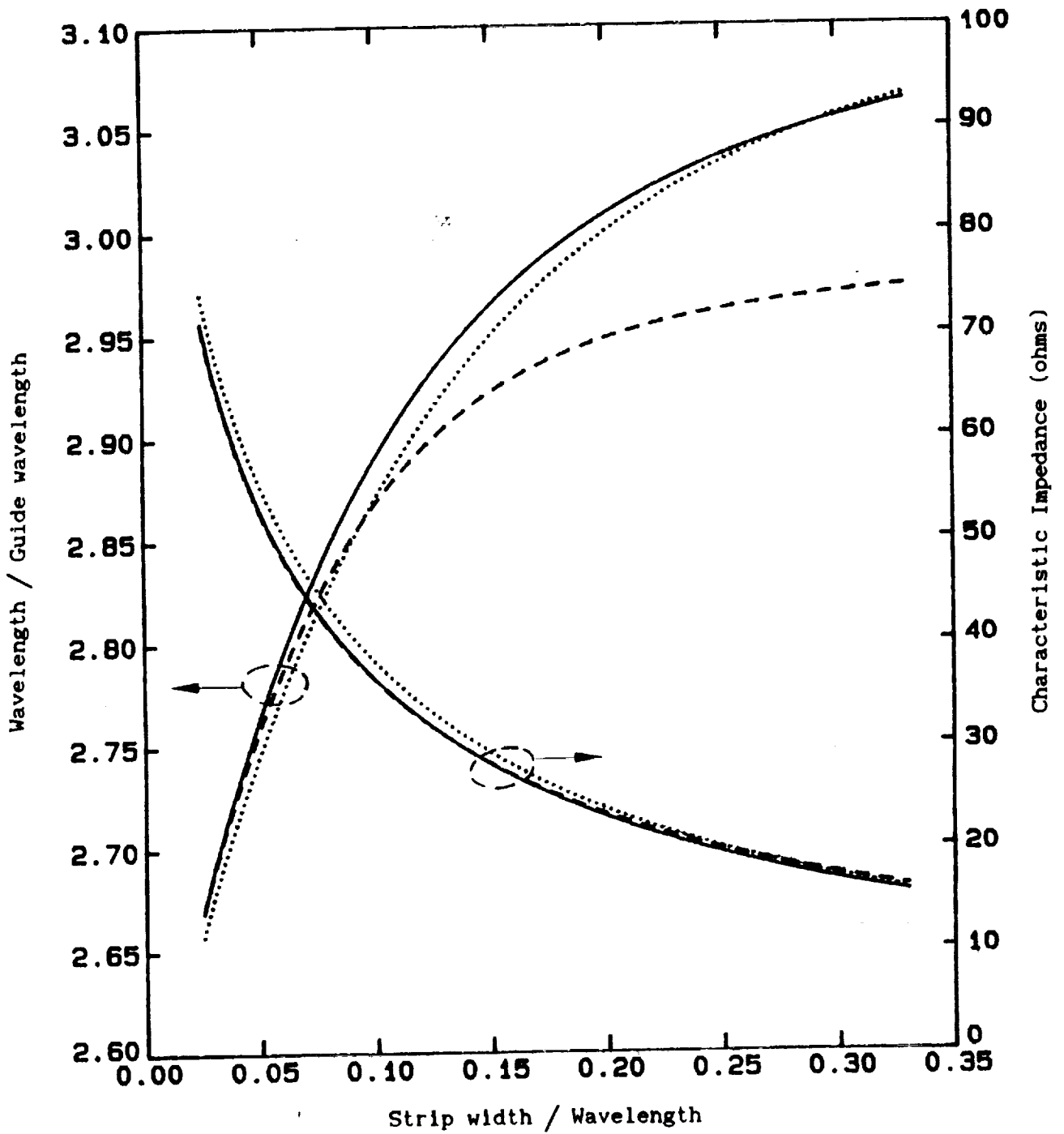


Figure 10. Characteristic impedance and normalized guide wavelength of microstrip line as a function of strip width.  
 $(\epsilon_r = 10, z' = d = 0.05 \lambda)$   
 (..... uniform, - - - nonuniform, ———  $N=25$  pulses).

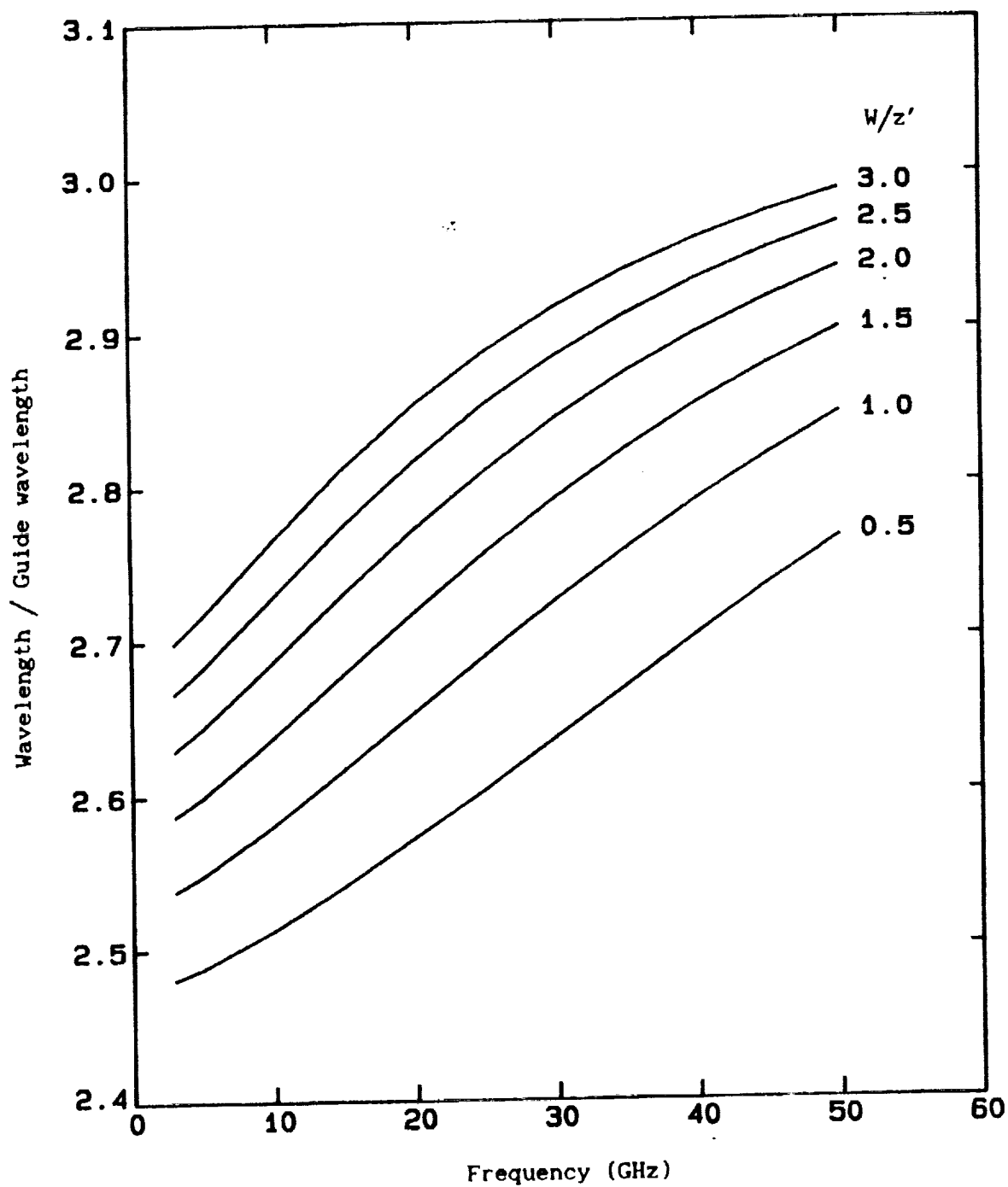


Figure 11a. Normalized guide wavelength of microstrip line as a function of frequency versus strip-width/strip-height. ( $\epsilon_r = 9.6$ ,  $z' = d = 0.06$  cm).

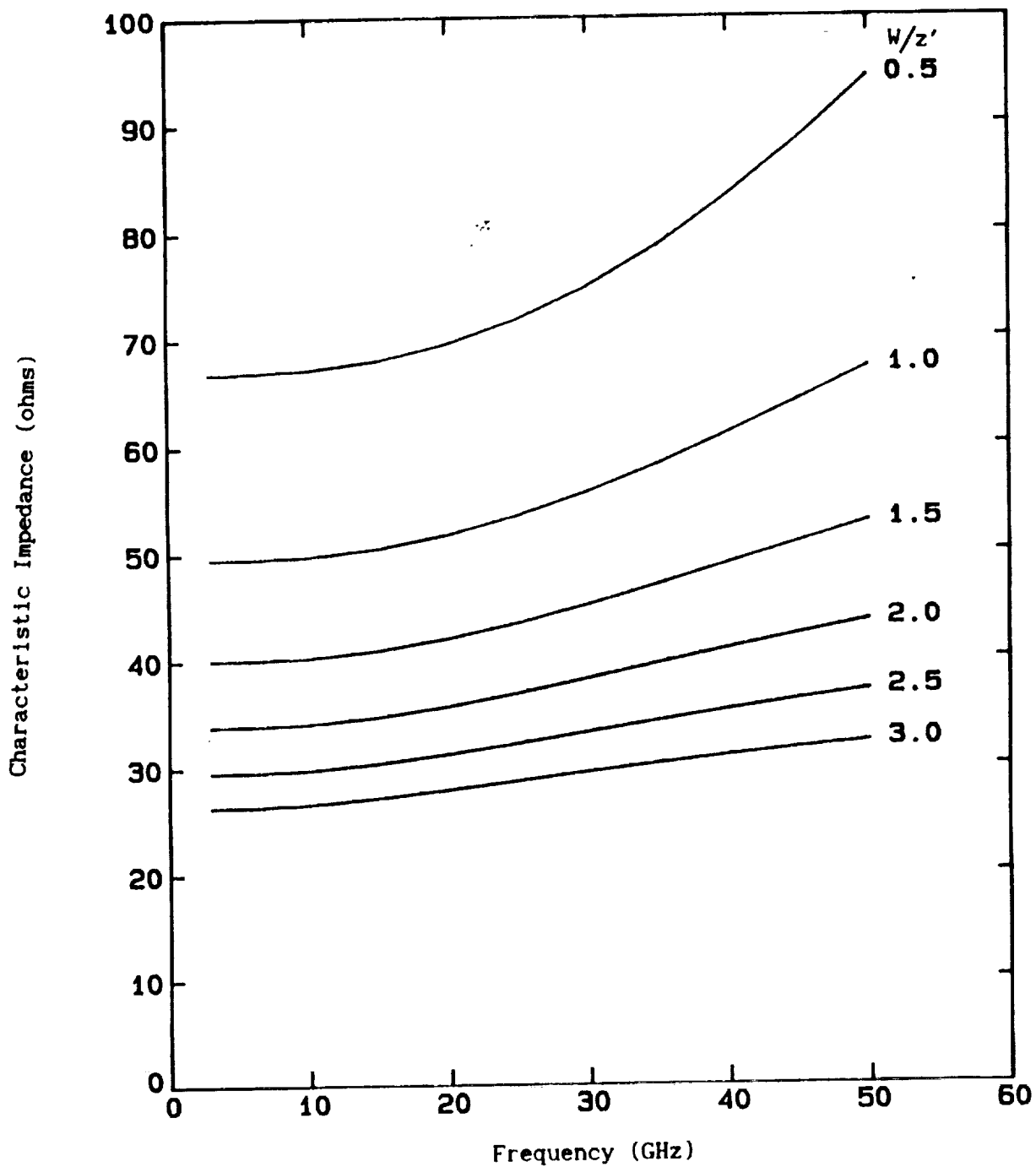


Figure 11b. Characteristic impedance of microstrip line as a function of frequency versus strip-width/strip-height. ( $\epsilon_r = 9.6$ ,  $z' = d = 0.06$  cm).

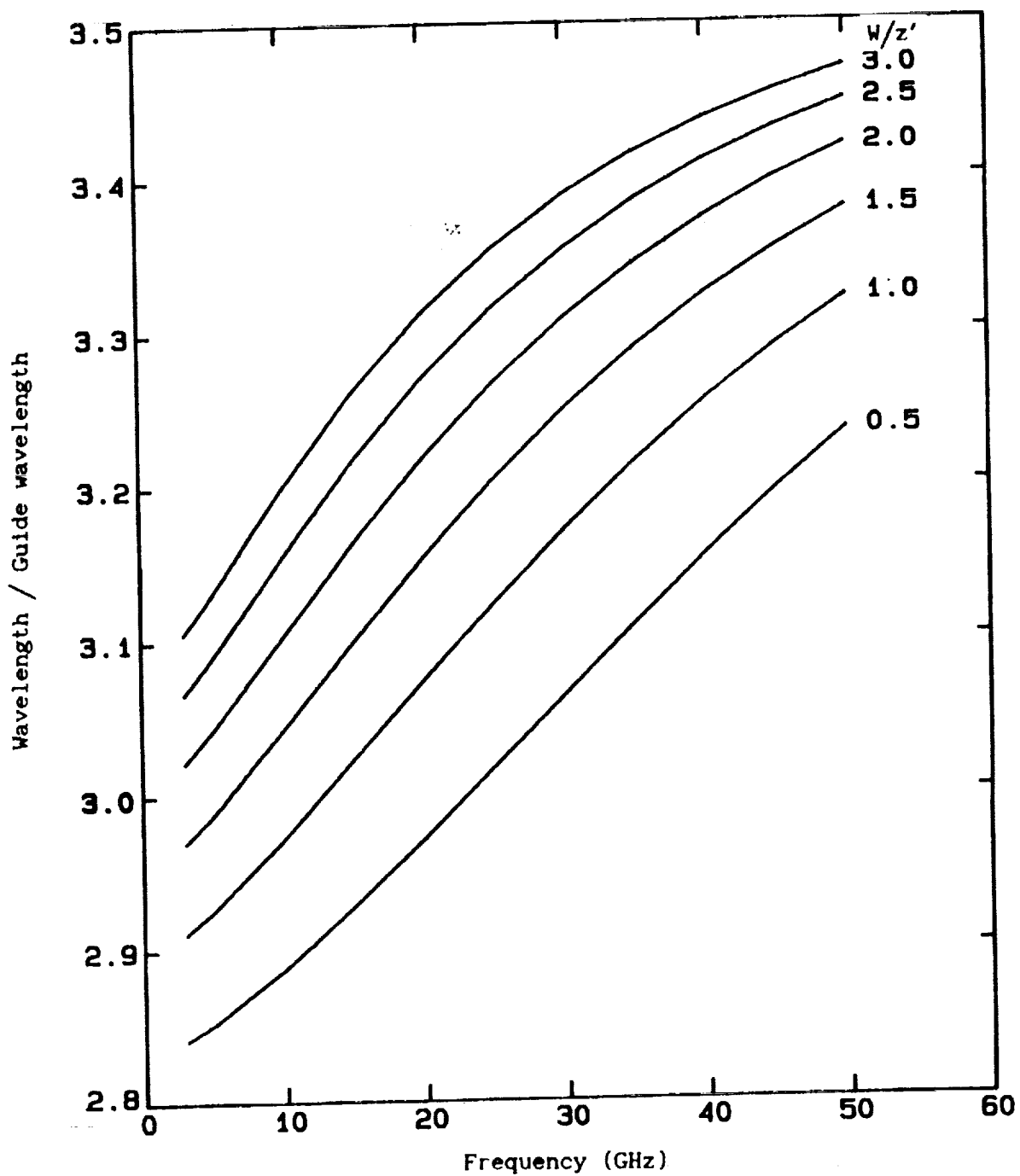


Figure 12a. Normalized guide wavelength of microstrip line as a function of frequency versus strip-width/strip-height. ( $\epsilon_r = 12.8$ ,  $z' = d = 0.06$  cm).

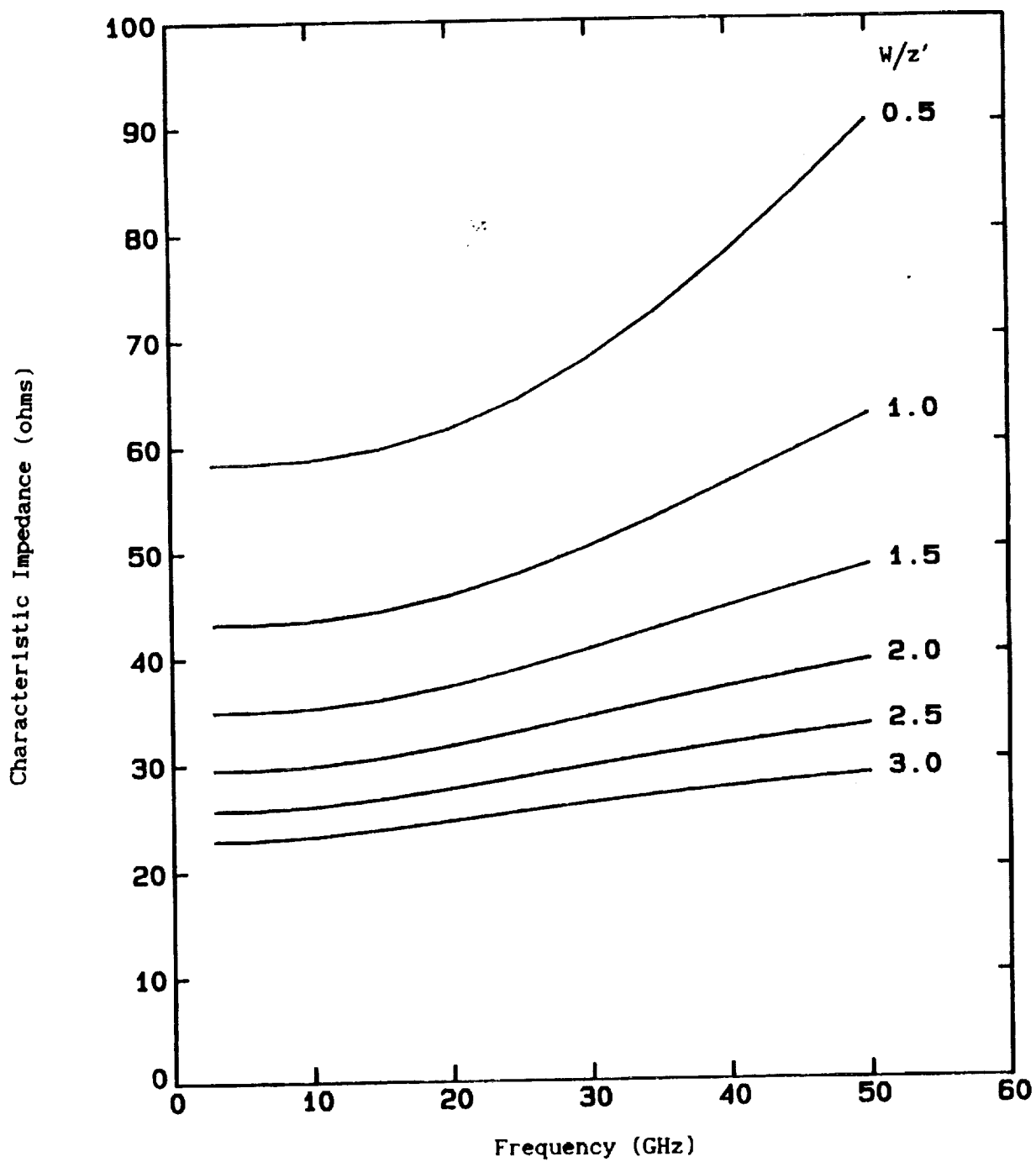


Figure 12b. Characteristic impedance of microstrip line as a function of frequency versus strip-width/strip-height. ( $\epsilon_r = 12.8$ ,  $z' = d = 0.06$  cm).

Appendix  
Instruction for using computer code

The main program MSTRIP.FOR calculates the characteristic impedance,  $Z_0$ , and the dominant mode propagation constant,  $K_0$ , of a microstrip line embedded in a grounded dielectric slab.

The main program must be linked to the following subroutines:

ZTEMP.FOR  
SRMN.FOR  
MATINV.FOR  
MATROW.FOR  
RSTRIP1.FOR  
RSTRIP.FOR  
MZO.FOR  
DFKX.FOR

INPUT DATA :

NU       = Integer selecting the type of current distribution.  
          1 for pulse distribution.  
          2 for uniform distribution.  
          3 for nonuniform distribution.

MM       = Number of pulses assumed in the pulse distribution.  
          (maximum number = 242).

ED       = Dielectric slab thickness in free space wavelengths.

EZP      = Height of strip above ground plane in free space wavelengths.

E        = Dielectric constant of slab.

E1       = Loss tangent of dielectric slab material.

WSTRIP   = Minimum microstrip width in free space wavelengths.

WMAX      = Maximum microstrip width in free space wavelengths.

OUTPUT DATA :

For pulse distribution, the transverse current distribution is output to file FOR010.DAT.

The characteristic impedance and dominant mode propagation constant as a function of microstrip width are output to file FOR013.DAT.

#### BRIEF OUTLINE OF METHOD :

The current distribution is assumed in the form;

$$J_x(x, y) = f(y) \exp(-JK_e x)$$

$$J_y(x, y) = 0.$$

For the above current distribution, the program calculates the dominant mode propagation constant and the characteristic impedance for one of three cases:

$$f(y) = I_0/W \quad (\text{uniform})$$

$$f(y) = \frac{2I_0}{\pi W \sqrt{1 - (2y/W)^2}} \quad (\text{nonuniform})$$

$$f(y) = I_n P_n(y) \quad n=1, 2, \dots, MM \quad (\text{pulses}).$$

For the distributions, the program calculates the  $K_e$  first by calling subroutine RSTRIP1. The subroutine RSTRIP1 solves equation 27 for  $K_e$ . RSTRIP1 uses standard technique to find zeros of a polynomial.

For the pulse distribution, the main program solves the matrix equation given in equation 36. This is done by computing mutual impedances by calling subroutine SRMN. The mutual impedance matrix is inverted using MATINV.  $I_n$  are found using MATROW. Equation 28 is then solved for  $K_e$  by using subroutine RSTRIP1.

The characteristic impedance for either distribution is computed by calling subroutine MZO.

```

      PROGRAM MSTRIP
C*****
C**
C**      This program calculates characteristic impedance of a
C**      microstrip line embedded in a dielectric slab.
C**
C**      This program gives characteristic impedance of a micro-
C**      strip line for three types of transverse current distri-
C**      butions. (1) Uniform, (2) Nonuniform with singularities
C**      at the strip edges, (3) General distribution with pulse
C**      approximations.
C**
C**      This program gives characteristic impedance of a micro-
C**      strip line as a function of strip width. It also gives
C**      the transverse current distribution.
C**
C*****
C      DIMENSION BTM(10),CZPAT(242),CZMN(242,242),CI(242)
C      1,CZT1(242,242),CZT2(242,242),CZT3(242,242),CVT(242)
C      2,CVT1(242),CI1(242),CI2(242)
C      COMPLEX CJ,CJE1,CZPAT,CZMN,ZPATR,CF1,CZT1,CZT2,CZT3,CVT,CVT1
C      COMMON CJ,CJE1,PI,TWOPI,PIO2,DELZ,DELD,DELDZ,DELW,XX,B,E,E1
C*****
C**
C**      *****      INPUT DATA      *****
C**
C**      NU = INTEGER AND SELECTS TYPE OF CURRENT DISTRIBUTION IN
C**      TRANSVERSE DIRECTION.
C**
C**      NU = 1 FOR PULSE DISTRIBUTION
C**
C**      = 2 FOR UNIFORM DISTRIBUTION
C**
C**      = 3 FOR NON-UNIFORM DISTRIBUTION
C**
C**      > or = 4 EXIT
C**
C**      MM = NUMBER OF PULSES FOR NU = 1
C**      = ANY NUMBER WHEN NU IS NOT EQUAL TO ONE.
C**      MAXIMUM VALUE OF MM MUST BE LESS THAN 242.
C**
C**      ED = DIELECTRIC SLAB THICKNESS IN WAVELENGTH
C**
C**      EZP = HIGHT OF MICROSTRIP FROM GROUND IN WAVELENGTH
C**      (EZP MUST BE LESS THAN ED)
C**
C**      E = DIELECTRIC CONSTANT OF SLAB.
C**
C**      WSTRIP = WIDTH OF MICROSTRIP IN WAVELENGTH
C**
C**      WMAX = MAXIMUM WIDTH OF STRIP IN WAVELENGTH
C**
C*****

```

```

WRITE(5,*) 'GIVE THE VALUE OF NU'
WRITE(5,*)'FOR PULSE DISTRIBUTION NU=1, FOR UNIFORM DISTRIBUTION
1NU=2, FOR NONUNIFORM DISTRIBUTION NU=3, NU GREATER THAN 3 EXIT.'
READ(5,*)NU
WRITE(5,*)'GIVE THE VALUE OF MM'
WRITE(5,*)'MM = 1 TO 242'
READ(5,*)MM
WRITE(5,*)'GIVE THE VALUE OF DIELECTRIC SLAB THICKNESS
1 IN WAVELENGTH'
READ(5,*)ED
WRITE(5,*)'GIVE THE POSTION OF STRIP FROM GROUND IN
1 WAVELENGTH'
READ(5,*)EZP
WRITE(5,*)'GIVE DIELECTRIC CONSTANT AND LOSS TANGENT
1 OF SLAB MATERIAL'
READ(5,*)E,E1
WRITE(5,*)'GIVE STRIP WIDTH, MAXIMUM STRIP WIDTH IN
1 WAVELENGTH'
READ(5,*)WSTRIP,WMAX
IF(NU.GT.3)GO TO 105
DELTX=0.001
CJ=(0.,1.)
PI=2.*ASIN(1.0)
PI02=PI*0.5
TWOPI=2.*PI
CJE1=CJ*(E-CJ*E*E1-1.)
SQRE=SQRT(E)
WRITE(13,*)'CHARACTERISTIC IMPEDANCE, PROPAGATION CONSTANT'
WRITE(13,*)'          OF MICROSTRIP LINE'
WRITE(13,*)'SLAB THICKNESS ED = ',ED
WRITE(13,*)'POSITION OF STRIP EZP = ',EZP
IF(NU.EQ.1)WRITE(13,*)'DISTRIBUTION PULSE WITH MM = ',MM
IF(NU.EQ.2)WRITE(13,*)'UNIFORM DISTRIBUTION'
IF(NU.EQ.3)WRITE(13,*)'NONUNIFORM DISTRIBUTION'
WRITE(13,*)'***STRIPWIDTH***','***PROPAGATION CONST.***', '***IMP.***'
103 DELD=TWOPI*ED
DELZ=TWOPI*EZP
DELW=TWOPI*WSTRIP
DELDZ=DELD-DELZ
DELWY=DELW/MM
B=0
C*****
C**                                     ****
C**      If dielectric constant of the slab = 1          ****
C**      then avoid surface wave calculation.            ****
C**                                     ****
C*****
IF(E.EQ.1.)GO TO 501
NE=1
NNMAX=1
CALL ZTEMP(BTM,NE,NNMAX,E,DELD)
B=BTM(1)

```

```

C*****
C**                                     ***
C**      If NU = 1 calculate mutual impedance matrix using ***
C**      subroutine SRMN.  When NU not equal to 1 mutual ***
C**      matrix is not required. ***
C**                                     ***
C*****
      IF(NU.NE.1)GO TO 502
501  CALL SRMN(MM,CZPAT)
      DO 10 I=1,MM
      DO 10 J=1,MM
      IF(J.LT.I)GO TO 10
      CZMN(I,J)=CZPAT(J-I+1)
10   CONTINUE
      DO 20 I=1,MM
      DO 20 J=1,MM
      IF(J.LE.I)GO TO 20
      CZMN(J,I)=CZMN(I,J)
20   CONTINUE
      DO 30 I=1,MM
      DO 30 J=1,MM
      CZT1(I,J)=CZMN(I,J)
30   CONTINUE
      EDK=SQRE*DELZ
      DKK=SQRE*DELD
      CF1=2.*CJ*SIN(EDK)*CEXP(CJ*DELZ)/(SQRE*COS(DKK)+CJ*SIN(DKK))
      DO 50 I=1,MM
      CVT(I)=-CF1
50   CONTINUE
      CALL MATINV(CZT1,MM,CZT2)
      CALL MATROW(CZT2,MM,CVT,CVT1)
      S1=0.
      DO 301 I=1,MM
      FACT=CABS(CVT1(I))
      S1=S1+FACT
301  CONTINUE
      DO 302 I=1,MM
      CVT1(I)=CVT1(I)/S1
302  CONTINUE
      DO 303 I=1,MM
      CI(I)=CABS(CVT1(I))
303  CONTINUE
C*****
C**                                     ***
C**      CI(I) is the current distribution in the transverse direction***
C**                                     ***
C*****
      WRITE(10,*) 'TRANSVERSE CURRENT DISTRIBUTION'
      WRITE(10,*) '***** Y/W *****', '*****CURRENT AMP.*****'
      DO 304 I=1,MM
      DWY=(2*I-1)
      DWY=DWY*DELWY*0.5/DELW
      WRITE(10,*)DWY,CI(I)
304  CONTINUE

```

```

C*****
C**
C** The transverse current distribution calculation for NU =1 ***
C** is completed here. The CI(I) as a function of DWY is ***
C** outputted at FOR010.DAT file. The CI(I) is not computed ***
C** when NU is not equal to one. ***
C**
C*****
502 NNMAX=1
C*****
C**
C** For the current distribution CI(I) the SUBROUTINE RSTRIP1 ****
C** calculates dominant mode propagation constant for the micro- *
C** strip line. ****
C** RSTRIP1 solves equation (17) for Ke. ****
C** RSTRIP1 must be linked to RTSRIP SUBROUTINE ****
C**
C*****
CALL RSTRIP1(NU, NNMAX, MM, DELWY, DELD, DELDZ, DELZ, DELW,
1E, E1, BTM, CI)
XX=BTM(1)
CALL MZO(NU, DELTX, ZPATR, CI, MM)
WRITE(13, *)WSTRIP, XX, ZPATR
WSTRIP=WSTRIP+0.005
IF(WSTRIP.GT.WMAX)GO TO 104
GO TO 103
105 WRITE(5, *) 'CURRENT DISTRIBUTION NOT SELECTED PROPERLY'
104 STOP
END

```

```

SUBROUTINE ZTEMP(BTM, NE, MAX, E, DK)
  DIMENSION BTM(1)
  SQRE=SQRT(E)
  DELTA=0.1
  B1=1.0
  B2=1.0+DELTA
  IF(B2.GT.SQRE)B2=SQRE
  BT=B2
  N=0
10  D1=SQRT(E-B1*B1)
    DK1=DK*D1
    S1=SIN(DK1)
    C1=COS(DK1)
    F1=E*SQRT(B1*B1-1.0)*C1-D1*S1
    D2=SQRT(ABS(E-B2*B2))
    DK2=DK*D2
    S2=SIN(DK2)
    C2=COS(DK2)
    F2=E*SQRT(B2*B2-1.0)*C2-D2*S2
    FT=F2
    IF(F1.LT.0.0.AND.F2.LT.0.0) GO TO 15
    IF(F1.GT.0.0.AND.F2.GT.0.0) GO TO 15
12  AF1=ABS(F1)
    AF2=ABS(F2)
    BP=B1+(B2-B1)*AF1/(AF1+AF2)
    IF(ABS(B2-B1).LE.1.E-04) GO TO 14
    DP=SQRT(E-BP*BP)
    DKP=DK*DP
    SP=SIN(DKP)
    CP=COS(DKP)
    FP=E*SQRT(BP*BP-1.0)*CP-DP*SP
    IF(ABS(FP).LE.1.E-04) GO TO 14
    IF(F1.LT.0.0.AND.FP.LT.0.0) GO TO 13
    IF(F1.GT.0.0.AND.FP.GT.0.0) GO TO 13
    B2=BP
    F2=FP
    GO TO 12
13  B1=BP
    F1=FP
    GO TO 12
14  N=N+1
    IF(N.GT.MAX) GO TO 17
    BTM(N)=BP
15  B1=BT
    IF(ABS(B1-SQRE).LE.1.E-05) GO TO 16
    B2=B1+DELTA
    IF(B2.GT.SQRE)B2=SQRE
    BT=B2
    GO TO 10
16  NE=N
17  RETURN
    END

```

```

SUBROUTINE SRMN(MM,ZPATR)
DIMENSION U1(3),U2(10),R1(3),R2(10),U(13),R(13)
DIMENSION BTM(1),SUMRR(242),ZPATR(242)
EQUIVALENCE (U1(1),U(1)),(U2(1),U(4)),(R1(1),R(1)),(R2(1),R(4))
COMMON CJ,CJE1,PI,TWOPI,PIO2,DELZ,DELD,DELDZ,DELW,XX,B,E,E1
COMPLEX CJ,CJE1,CF1,SUMRR,ZPATR,CCFA
DATA U1/0.11270166537925,.5,0.88729833462074/,U2/.01304673574141
A,.06746831665550,.16029521585048,.28330230293537,.42556283050918
B,.57443716949081,.71669769706462,.83970478414951,.93253168334449
C,.98695326425858/,R1/.277777777777777,.444444444444444,.2777777777
D77777/,R2/.03333567215434,.07472567457529,.10954318125799,.134633
E35965499,.14776211235737,.14776211235737,.13463335965499,.109543
F18125799,.07472567457529,.03333567215434/
DELWY=DELW/MM
SQRE=SQRT(E)
DWKO2=0.5*DELWY
CONST=377.7*DELZ*DELWY/PI
DO 201 I=1,MM
SUMRR(I)=(0.,0.)
ZPATR(I)=(0.,0.)
201 CONTINUE
NF=1
NQ=5
XX1=0.0001
XX2=1.-0.0001
100 DELT=(XX2-XX1)/FLOAT(NQ)
DO 80 K=1,NQ
XI=K-1
FF=XX1+XI*DELT
DO 80 JL=4,13
UU=U(JL)*DELT+FF
BSQ=UU*UU
BET=UU
BETA=UU
DWKB2=BETA*DWKO2
AKX1=BSQ
IF(E.EQ.1.)GO TO 501
IF(AKX1-1.)10,10,12
10 B1=SQRT(1.-AKX1)
BE=SQRT(E-AKX1)
AZ=BE*DELZ
AD=DELD*BE
ADZ=DELDZ*BE
SINZ=SIN(AZ)
COSD=COS(AD)
SIND=SIN(AD)
COSDZ=COS(ADZ)
SINDZ=SIN(ADZ)
SINZ1=1.0
IF(ABS(AZ).LT.1.E-05)GO TO 21
SINZ1=SINZ/AZ
21 CF1=SINZ1*(BE*COSDZ+CJ*B1*SINDZ)/(BE*COSD+CJ*B1*SIND)
GO TO 20
12 IF(AKX1-E)14,14,15

```

```

14  B1=SQRT(AKX1-1.)
    BE=SQRT(E-AKX1)
    AZ=BE*DELZ
    AD=BE*DELD
    ADZ=BE*DELDZ
    SINZ=SIN(AZ)
    COSD=COS(AD)
    SIND=SIN(AD)
    COSDZ=COS(ADZ)
    SINDZ=SIN(ADZ)
    SINZ1=1.
    IF(ABS(AZ).LT.1.E-05)GO TO 22
    SINZ1=SINZ/AZ
22  CF1=SINZ1*(BE*COSDZ+B1*SINDZ)/(BE*COSD+B1*SIND)
    GO TO 20
15  B1=SQRT(AKX1-1.)
    BE=SQRT(AKX1-E)
    AZ=DELZ*BE
    AD=DELD*BE
    ADZ=DELDZ*BE
    EP1=0.0
    IF(ABS(AZ).GT.20.)GO TO 23
    EP1=EXP(-2.*AZ)
23  EPD=0.
    IF(ABS(AD).GT.20.)GO TO 24
    EPD=EXP(-2.*AD)
24  EPDZ=0.
    IF(ABS(ADZ).GT.20.)GO TO 25
    EPDZ=EXP(-2.*ADZ)
25  CF1=(1.-EP1)*(BE*(1.+EPDZ)+B1*(1.-EPDZ))
    CF1=CF1/(BE*(1.+EPD)+B1*(1.-EPD))
    CF1=CF1/(2.*AZ)
    GO TO 20
501 IF(AKX1-1.)502,502,503
502 BE=SQRT(1.-AKX1)
    AZ=BE*DELZ
    SINZ=SIN(AZ)
    IF(ABS(AZ).LT.1.E-05)GO TO 504
    SINZ1=SINZ/AZ
504 CF1=SINZ1*(COS(AZ)-CJ*SINZ)
    GO TO 20
503 BE=SQRT(AKX1-1.)
    AZ=BE*DELZ
    EZ=0.
    IF(ABS(AZ).GT.20.)GO TO 505
    EZ=EXP(-2.*AZ)
505 CF1=(1.-EZ)/(2.*AZ)
20  DWKA2=DWKB2
    F1=1.0
    IF(ABS(DWKA2).LT.1.E-05)GO TO 301
    F1=SIN(DWKA2)/DWKA2
301 CCFA=-CJ*CONST*CF1
    CCFA=CCFA*F1*F1
    DO 202 I=1,MM

```

```

SUMRR(I)=SUMRR(I)+CCFA*R(JL)*COS(2.*DWKA2*(1-I))
202 CONTINUE
80 CONTINUE
DO 203 I=1,MM
SUMRR(I)=SUMRR(I)*DELT
203 CONTINUE
DO 204 I=1,MM
ZPATR(I)=ZPATR(I)+SUMRR(I)
204 CONTINUE
DO 205 I=1,MM
SUMRR(I)=(0.,0.)
205 CONTINUE
CPOLE=0.0001
NF=NF+1
IF(NF.EQ.2)XX1=1.+CPOLE
IF(NF.EQ.2)XX2=SQRE-CPOLE
IF(NF.EQ.2)GO TO 100
IF(NF.EQ.3)XX1=SQRE+CPOLE
IF(NF.EQ.3)XX2=XX1+1.
IF(NF.EQ.3)GO TO 100
IF(XX2.LT.5.0.AND.XX2.GT.0.)DINC=1.0
IF(XX2.LT.25.0.AND.XX2.GT.5.0)DINC=5.
IF(XX2.LT.100.0.AND.XX2.GT.25.0)DINC=25.0
IF(XX2.GT.100.0)DINC=100.0
XX1=XX2
XX2=XX1+DINC
IF(XX2.LT.2000.0)GO TO 100
RETURN
END

```

```

SUBROUTINE MATINV(SUM1,N,SUM2)
DIMENSION IST(242),JST(242)
COMPLEX SUM1(242,242),SUM2(242,242)
DO 1 I=1,N
  IST(I)=0
1  JST(I)=0
  DO 30 IND=1,N
    H=0.0
    DO 10 I=1,N
      DO 3 KCH=1,N
        IF(IST(KCH).EQ.I)GO TO 10
3     CONTINUE
        DO 9 J=1,N
          DO 4 KCH=1,N
            IF(JST(KCH).EQ.J) GO TO 9
4         CONTINUE
            IF(H.GE.CABS(SUM1(I,J))) GO TO 9
            H=CABS(SUM1(I,J))
            IM=I
            JM=J
9         CONTINUE
10        CONTINUE
            IST(IND)=IM
            JST(IND)=JM
            DO 20 I=1,N
              DO 20 J=1,N
                IF(I-IM)5,6,5
6              SUM2(I,J)=-SUM1(I,J)/SUM1(IM,JM)
              GO TO 20
5              IF(J-JM)7,8,7
8              SUM2(I,J)=SUM1(I,J)/SUM1(IM,JM)
              GO TO 20
7              SUM2(I,J)=SUM1(I,J)-SUM1(I,JM)*SUM1(IM,J)/SUM1(IM,JM)
20         CONTINUE
            SUM2(IM,JM)=1./SUM1(IM,JM)
            DO 25 I=1,N
              DO 25 J=1,N
25          SUM1(I,J)=SUM2(I,J)
30         CONTINUE
            DO 35 I=1,N
              IF(IST(I).EQ.JST(I))GO TO 35
              DO 45 J=1,N
45          SUM2(JST(I),J)=SUM1(IST(I),J)
35         CONTINUE
            DO 26 I=1,N
              DO 26 J=1,N
26          SUM1(I,J)=SUM2(I,J)
            DO 50 I=1,N
              IF(IST(I).EQ.JST(I))GO TO 50
              DO 55 J=1,N
55          SUM2(J,IST(I))=SUM1(J,JST(I))
50         CONTINUE
            RETURN
            END

```

```

SUBROUTINE MATROW(SUM2, N, SUM, SUM5)
COMPLEX SUM2(242, 242), SUM(242), SUM5(242)
DO 110 I=1, N
SUM5(I)=(0., 0.)
110 CONTINUE
DO 120 I=1, N
DO 120 J=1, N
SUM5(I)=SUM2(I, J)*SUM(J)+SUM5(I)
120 CONTINUE
RETURN
END

```

```

SUBROUTINE RSTRIP1(NU, NNMAX, MM, DELWY, DK, DZP, ZPK, DWSTRIP,
1E, E1, BTM, CI)
  DIMENSION BTM(1), CI(242)
  COMPLEX ZPATRR
  SQRE=SQRT(E)
  N=0
  DELTA=0.1
  B1=SQRE
  B2=B1-DELTA
10  CALL RSTRIP(NU, MM, DELWY, DK, DZP, ZPK, DWSTRIP, E, E1, B1, ZPATRR, CI)
  F1=REAL(ZPATRR)
  CALL RSTRIP(NU, MM, DELWY, DK, DZP, ZPK, DWSTRIP, E, E1, B2, ZPATRR, CI)
  F2=REAL(ZPATRR)
  FT=F2
  IF(F1.LT.0.0.AND.F2.LT.0.0) GO TO 15
  IF(F1.GT.0.0.AND.F2.GT.0.0) GO TO 15
12  AF1=ABS(F1)
  AF2=ABS(F2)
  BP=B1-(B1-B2)*AF1/(AF1+AF2)
  IF(ABS(B2-B1).LE.1.E-04) GO TO 14
  CALL RSTRIP(NU, MM, DELWY, DK, DZP, ZPK, DWSTRIP, E, E1, BP, ZPATRR, CI)
  FP=REAL(ZPATRR)
  IF(ABS(FP).LE.1.E-04) GO TO 14
  IF(F1.LT.0.0.AND.FP.LT.0.0) GO TO 13
  IF(F1.GT.0.0.AND.FP.GT.0.0) GO TO 13
  B2=BP
  F2=FP
  GO TO 12
13  B1=BP
  F1=FP
  GO TO 12
14  IF(ABS(FP).LT.1.E-04)GO TO 19
  GO TO 15
19  N=N+1
  BTM(N)=BP
  IF(N.GE.1)GO TO 17
15  B1=B2
  B2=B1-DELTA
  GO TO 10
16  NE=N
17  RETURN
  END

```

```

SUBROUTINE RSTRIP(NU,MM,DELWY,DK,DZP,ZPK,DWSTRIP,E,E1,
1          XX,ZPATR,CI)
  DIMENSION U1(3),U2(10),R1(3),R2(10),U(13),R(13),BTM(1),CI(242)
  EQUIVALENCE (U1(1),U(1)),(U2(1),U(4)),(R1(1),R(1)),(R2(1),R(4))
  COMPLEX CJ,CJE1,CF1,CF2,SUMRR,ZPATR,CCFA
  DATA U1/0.11270166537925,.5,0.88729833462074/,U2/.01304673574141
A,.06746831665550,.16029521585048,.28330230293537,.42556283050918
B,.57443716949081,.71669769706462,.83970478414951,.93253168334449
C,.98695326425858/,R1/.2777777777777777,.4444444444444444,.2777777777
D7777/,R2/.03333567215434,.07472567457529,.10954318125799,.134633
E35965499,.14776211235737,.14776211235737,.13463335965499,.109543
F18125799,.07472567457529,.03333567215434/
  CJ=(0.,1.)
  PI=2.*ASIN(1.0)
  TWOPI=2.*PI
  PI02=PI/2.
  SQRE=SQRT(E)
  DWK02=0.5*DWSTRIP
  DWKY2=0.5*DELWY
  CJE1=CJ*(E-CJ*E1*E-1.)
  SUMRR=(0.,0.)
  ZPATR=(0.,0.)
  CONTINUE
  NF=1
  NQ=5
  XX1=0.0001
  XX2=1.-0.0001
100 DELT=(XX2-XX1)/FLOAT(NQ)
  DO 80 K=1,NQ
    XI=K-1
    FF=XX1+XI*DELT
    DO 80 JL=4,13
      UU=U(JL)*DELT+FF
      BSQ=UU*UU
      BET=UU
      BETA=UU
      DWKB2=BETA*DWK02
      DYKB2=BETA*DWKY2
      AKX1=XX*XX+BSQ
      IF(AKX1-1.)10,10,12
10  B1=SQRT(1.-AKX1)
      BE=SQRT(E-AKX1)
      AZ=BE*ZPK
      AD=DK*BE
      ADZ=DZP*BE
      SINZ=SIN(AZ)
      COSD=COS(AD)
      SIND=SIN(AD)
      COSDZ=COS(ADZ)
      SINDZ=SIN(ADZ)
      SINZ1=1.0
      IF(ABS(AZ).LT.1.E-05)GO TO 21
      SINZ1=SINZ/AZ
21  CF1=SINZ1*(BE*COSDZ+CJ*B1*SINDZ)/(BE*COSD+CJ*B1*SIND)

```

```

CF2=SINZ1*SINZ*BE*BE/(BE*COSD+CJ*B1*SIND)
CF2=CF2/(E*B1*COSD+CJ*BE*SIND)
GO TO 20
12 IF(AKX1-E)14,14,15
14 B1=SQRT(AKX1-1.)
BE=SQRT(E-AKX1)
AZ=BE*ZPK
AD=BE*DK
ADZ=BE*DZP
SINZ=SIN(AZ)
COSD=COS(AD)
SIND=SIN(AD)
COSDZ=COS(ADZ)
SINDZ=SIN(ADZ)
SINZ1=1.
IF(ABS(AZ).LT.1.E-05)GO TO 22
SINZ1=SINZ/AZ
22 CF1=SINZ1*(BE*COSDZ+B1*SINDZ)/(BE*COSD+B1*SIND)
CF2=SINZ1*SINZ*BE*BE/(BE*COSD+B1*COSD)
CF2=CJ*CF2/(B1*E*COSD-BE*SIND)
GO TO 20
15 B1=SQRT(AKX1-1.)
BE=SQRT(AKX1-E)
AZ=ZPK*BE
AD=DK*BE
ADZ=DZP*BE
EP1=0.0
IF(ABS(AZ).GT.20.)GO TO 23
EP1=EXP(-2.*AZ)
23 EPD=0.
IF(ABS(AD).GT.20.)GO TO 24
EPD=EXP(-2.*AD)
24 EPDZ=0.
IF(ABS(ADZ).GT.20.)GO TO 25
EPDZ=EXP(-2.*ADZ)
25 CF1=(1.-EP1)*(BE*(1.+EPDZ)+B1*(1.-EPDZ))
CF1=CF1/(BE*(1.+EPD)+B1*(1.-EPD))
CF1=CF1/(2.*AZ)
CF2=-CJ*BE*BE*(1.-EP1)*(1.-EP1)*EPDZ/AZ
CF2=CF2/(BE*(1.+EPD)+B1*(1.-EPD))
CF2=CF2/(E*B1*(1.+EPD)+BE*(1.-EPD))
20 DWKA2=DWKB2
IF(NU.NE.1)GO TO 501
F11=SIN(DYKB2)
F1=1.
IF(ABS(DWKA2).LT.1.E-05)GO TO 301
F1=SIN(DWKA2)/DWKA2
301 F2=1.
IF(ABS(DYKB2).LT.1.E-05)GO TO 302
F2=F11/DYKB2
302 GO TO 508
501 F3=0.5*DWKA2
IF(ABS(F3).LT.1.E-05)GO TO 506
F3=SIN(F3)/F3

```

```

506 F4=1.
   IF (ABS(DWKA2).LT.1.E-05)GO TO 507
   F4=SIN(DWKA2)/DWKA2
507 F1=1.
   F2=1.
   IF (ABS(DWKA2).LT.1.E-05)GO TO 503
   IF (NU.EQ.2)F2=F4
   IF (NU.EQ.3)CALL BJOR(DWKA2,F2,IERR)
   IF (NU.EQ.4)F2=2.*F4-F3*F3
503 F2=F2*F2
508 CCFA=((E-XX*XX)*CF1-CJE1*XX*XX*CF2)
   CCFA=CCFA*F1*F2
   IF (NU.NE.1)GO TO 504
   S1=0.
   DO 303 I=1,MM
   XARG=MM/2
   XARG1=I
   XARG2=0.5
   XARG=XARG-(XARG1-XARG2)
   FACT=CI(I)*COS(2.*DYKB2*XARG)
   S1=S1+FACT
303 CONTINUE
   GO TO 505
504 S1=1.
505 SUMRR=SUMRR+CCFA*R(JL)*S1
80 CONTINUE
   SUMRR=SUMRR*DELT
   ZPATR=ZPATR+SUMRR
   SUMRR=(0.,0.)
   CPOLE=0.0001
   NF=NF+1
   IF (NF.EQ.2)XX1=1.+CPOLE
   IF (NF.EQ.2)XX2=SQRE-CPOLE
   IF (NF.EQ.2)GO TO 100
   IF (NF.EQ.3)XX1=SQRE+CPOLE
   IF (NF.EQ.3)XX2=XX1+1.
   IF (NF.EQ.3)GO TO 100
   IF (XX2.LT.5.0.AND.XX2.GT.0.)DINC=1.0
   IF (XX2.LT.25.0.AND.XX2.GT.5.0)DINC=5.
   IF (XX2.LT.100.0.AND.XX2.GT.25.0)DINC=25.0
   IF (XX2.GT.100.0)DINC=100.0
   XX1=XX2
   XX2=XX1+DINC
   IF (XX2.LT.1000.0)GO TO 100
   RETURN
   END

```

```

      SUBROUTINE MZO(NU,DELTX,ZPATR,CI,MM)
C*****
C**  This program computes the characteristic impedance of micro- **
C**  line. This program calls subroutine DFKX to calculate **
C**  derivatives of the integrand appearing in equations (12), **
C**  (13), and (14). **
C** **
C** **
C**  INPUT PARAMETERS: **
C** **
C**  NU = 1 FOR PULSE DISTRIBUTION **
C**      = 2 FOR UNIFORM DISTRIBUTION **
C**      = 3 FOR NONUNIFORM DISTRIBUTION **
C** **
C**  DELTX = A CONSTANT IN THE RANGE 0.01-0.001 **
C** **
C**  CI = CURRENT DISTRIBUTION AMP. **
C**      IT IS ZERO IF NU=2 OR NU=3 **
C** **
C**  MM = NUMBER OF PULES **
C** **
C**  OUTPUT PARAMETERS: **
C** **
C**  ZPATR = CHARACTERISTIC IMPEDANCE OF MICROSTRIP LINE **
C** **
C*****
      DIMENSION U1(3),U2(10),R1(3),R2(10),U(13),R(13)
      DIMENSION BTM(1),CI(242)
      EQUIVALENCE (U1(1),U(1)),(U2(1),U(4)),(R1(1),R(1)),(R2(1),R(4))
      COMMON CJ,CJE1,PI,TWOPI,PIO2,DELZ,DELD,DELDZ,DELW,XX,B,E,E1
      COMPLEX CJ,CJE1,CF1,CF2,SUMRR,ZPATR,CCFA,DF
      DATA U1/0.11270166537925,.5,0.88729833462074/,U2/.01304673574141
      A,.06746831665550,.16029521585048,.28330230293537,.42556283050918
      B,.57443716949081,.71669769706462,.83970478414951,.93253168334449
      C,.98695326425858/,R1/.2777777777777777,.4444444444444444,.2777777777
      D7777/,R2/.03333567215434,.07472567457529,.10954318125799,.134633
      E35965499,.14776211235737,.14776211235737,.13463335965499,.109543
      F18125799,.07472567457529,.03333567215434/
      CPOLE=0.0001
      SQRE=SQRT(E)
      DELWY=DELW/MM
      DWK02=0.5*DELW
      DYK02=0.5*DELWY
      CONST=-377.7*DELZ/(PI*E)
      SUMRR=(0.,0.)
      ZPATR=(0.,0.)
      NF=1
      NQ=5
      XX1=0.0001
      XX2=1.-0.0001
100 DELT=(XX2-XX1)/FLOAT(NQ)
      DO 80 K=1,NQ
      XI=K-1
      FF=XX1+XI*DELT

```

```

DO 80 JL=4,13
UU=U(JL)*DELT+FF
BSQ=UU*UU
BET=UU
BETA=UU
DWKB2=BETA*DWK02
DYKB2=BETA*DYK02
CALL DFKX(XX, DELTX, DELD, DELDZ, DELZ, E, CJ, CJE1, BETA, DF)
AKX1=XX*XX+BSQ
IF(AKX1-1.)10,10,12
10 B1=SQRT(1.-AKX1)
BE=SQRT(E-AKX1)
AD=DELD*BE
AZ=DELZ*BE
ADZ=DELDZ*BE
SINZ=SIN(AZ)
SIND=SIN(AD)
COSD=COS(AD)
SINDZ=SIN(ADZ)
COSDZ=COS(ADZ)
SINZ1=1.
IF(ABS(AZ).LT.1.E-05)GO TO 201
SINZ1=SINZ/AZ
201 CF1=SINZ1*(BE*COSDZ+CJ*B1*SINDZ)
CF1=CF1/(BE*COSD+CJ*B1*SIND)
CF2=BE*BE*SINZ1*SINZ/(BE*COSD+CJ*B1*SIND)
CF2=CF2/(E*B1*COSD+CJ*BE*SIND)
GO TO 20
12 IF(AKX1-E)14,14,18
14 B1=SQRT(AKX1-1.)
BE=SQRT(E-AKX1)
AD=DELD*BE
AZ=DELZ*BE
ADZ=DELDZ*BE
SINZ=SIN(AZ)
SIND=SIN(AD)
COSD=COS(AD)
SINDZ=SIN(ADZ)
COSDZ=COS(ADZ)
SINZ1=1.
IF(ABS(AZ).LT.1.E-05)GO TO 202
SINZ1=SINZ/AZ
202 CF1=SINZ1*(BE*COSDZ+B1*SINDZ)/(BE*COSD+B1*SIND)
CF2=CJ*BE*BE*SINZ1*SINZ/(BE*COSD+B1*SIND)
CF2=CF2/(E*B1*COSD-BE*SIND)
GO TO 20
18 B1=SQRT(AKX1-1.)
BE=SQRT(AKX1-E)
AD=DELD*BE
AZ=DELZ*BE
ADZ=DELDZ*BE
AZ2=2.*AZ
AD2=2.*AD
ADZ2=2.*ADZ

```

```

EZ=0.
IF(ABS(AZ2).GT.15.)GO TO 203
EZ=EXP(-AZ2)
203 EDD=0.
IF(ABS(AD2).GT.15.)GO TO 204
EDD=EXP(-AD2)
204 EDZ=0.
IF(ABS(ADZ2).GT.15.)GO TO 205
EDZ=EXP(-ADZ2)
205 CF1=(1.-EZ)*(BE*(1.+EDZ)+B1*(1.-EDZ))
CF1=CF1/(BE*(1.+EDD)+B1*(1.-EDD))
CF1=CF1/AZ2
CF2=BE*BE*(1.-EZ)*(1.-EZ)*EDZ/AZ
CF2=-CJ*CF2/(BE*(1.+EDD)+B1*(1.-EDD))
CF2=CF2/(E*B1*(1.+EDD)+BE*(1.-EDD))
20 IF(NU.NE.1)GO TO 501
S1=0.
DO 302 I=1,MM
DO 302 J=1,MM
XARG=(I-J)
FACT=CI(I)*CI(J)*COS(2.*DYKB2*XARG)
S1=S1+FACT
302 CONTINUE
F2=1.
IF(ABS(DYKB2).LT.1.E-05)GO TO 303
F2=SIN(DYKB2)/DYKB2
303 GO TO 502
501 S1=1.
F3=0.5*DWKB2
IF(ABS(F3).LT.1.E-05)GO TO 507
F3=SIN(F3)/F3
507 F2=1.
IF(ABS(DWKB2).LT.1.E-05)GO TO 508
F2=SIN(DWKB2)/DWKB2
508 IF(NU.EQ.2)F2=F2
IF(NU.EQ.3)CALL BJOR(DWKB2,F2,IERR)
IF(NU.EQ.4)F2=2.*F2-F3*F3
502 CCFA=((E-XX*XX)*CF1-CJE1*XX*XX*CF2-XX*DF)/XX
CCFA=CONST*DF*F2*F2*S1
SUMRR=SUMRR+CCFA*R(JL)
80 CONTINUE
SUMRR=SUMRR*DELT
ZPATR=ZPATR+SUMRR
SUMRR=(0.,0.)
NF=NF+1
IF(NF.EQ.2)XX1=1.+CPOLE
IF(NF.EQ.2)XX2=B-CPOLE
IF(NF.EQ.2)GO TO 100
IF(NF.EQ.3)XX1=B+CPOLE
IF(NF.EQ.3)XX2=SQRE-CPOLE
IF(NF.EQ.3)GO TO 100
IF(NF.EQ.4)XX1=SQRE+CPOLE
IF(NF.EQ.4)XX2=XX2+1.0
IF(NF.EQ.4)GO TO 100

```

```
IF (XX2. LT. 5. 0. AND. XX2. GT. 0. ) DINC=1. 0
IF (XX2. LT. 25. 0. AND. XX2. GT. 5. 0) DINC=5.
IF (XX2. LT. 100. 0. AND. XX2. GT. 25. 0) DINC=25. 0
IF (XX2. GT. 100. 0) DINC=100. 0
XX1=XX2
XX2=XX1+DINC
IF (XX2. LT. 3000. 0) GO TO 100
ZPATR=ZPATR/4.
RETURN
END
```

```

      SUBROUTINE DFKX(XX, DELTX, DK, DZP, ZPK, E, CJ, CJE1, BETA, DF)
C*****
C**
C**   This program calculates the derivative of integrand
C**   appearing in equations (12), (13), and (14).
C**
C*****
      DIMENSION DF1(2)
      COMPLEX DF, DF1
      COMPLEX CJ, CJE1, CF1, CF2
      AKX1=XX-DELTX
      I=1
      BSQ=BETA*BETA
100  AB1=BSQ+AKX1*AKX1
      IF(AB1-1.)10,10,12
10   B1=SQRT(1.-AB1)
      BE1=SQRT(E-AB1)
      AD=BE1*DK
      AZ=BE1*ZPK
      ADZ=BE1*ADZ
      SINZ=SIN(AZ)
      SIND=SIN(AD)
      COSD=COS(AD)
      SINDZ=SIN(ADZ)
      COSDZ=COS(ADZ)
      SINZ1=1.0
      IF(ABS(AZ).LT.1.E-05)GO TO 21
      SINZ1=SINZ/AZ
21   CF1=SINZ1*(BE1*COSDZ+CJ*B1*SINDZ)/(BE1*COSD+CJ*B1*SIND)
      CF2=SINZ1*SINZ*BE1*BE1/(BE1*COSD+CJ*B1*SIND)
      CF2=CF2/(E*B1*COSD+CJ*BE1*SIND)
      GO TO 20
12  IF(AB1-E)14,14,15
14   B1=SQRT(AB1-1.)
      BE1=SQRT(E-AB1)
      AD=BE1*DK
      AZ=BE1*ZPK
      ADZ=BE1*DZP
      SINZ=SIN(AZ)
      SIND=SIN(AD)
      COSD=COS(AD)
      SINDZ=SIN(ADZ)
      COSDZ=COS(ADZ)
      SINZ1=1.
      IF(ABS(AZ).LT.1.E-05)GO TO 22
      SINZ1=SINZ/AZ
22   CF1=SINZ1*(BE1*COSDZ+B1*SINDZ)/(BE1*COSD+B1*SIND)
      CF2=SINZ1*SINZ*BE1*BE1/(BE1*COSD+B1*SIND)
      CF2=CJ*CF2/(E*B1*COSD-BE1*SIND)
      GO TO 20
15  B1=SQRT(AB1-1.)
      BE1=SQRT(AB1-E)
      AZ=BE1*ZPK
      AD=BE1*DK

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```

ADZ=BE1*DZP
EP1=0.0
IF (ABS(AZ).GT.20.)GO TO 23
EP1=EXP(-2.*AZ)
23 EPD=0.0
   IF (ABS(AD).GT.20.)GO TO 24
   EPD=EXP(-2.*AD)
24 EPDZ=0.0
   IF (ABS(ADZ).GT.20.)GO TO 25
   EPDZ=EXP(-2.*ADZ)
25 CF1=(1.-EP1)*(BE1*(1.+EPDZ)+B1*(1.-EPDZ))
   CF1=CF1/(BE1*(1.+EPD)+B1*(1.-EPD))
   CF1=CF1/(2.*AZ)
   CF2=-CJ*BE1*BE1*(1.-EP1)*(1.-EP1)*EPDZ/AZ
   CF2=CF2/(BE1*(1.+EPD)+B1*(1.-EPD))
   CF2=CF2/(E*B1*(1.+EPD)+BE1*(1.-EPD))
20 DF1(I)=(E-AKX1*AKX1)*CF1-CJE1*AKX1*AKX1*CF2
   AKX1=XX+DELTX
   I=I+1
   IF (I.LE.2)GO TO 100
   DF=(DF1(2)-DF1(1))/DELTX
RETURN
END

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## Report Documentation Page

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16. Abstract  The dyadic Green's function for a current embedded in a grounded dielectric slab is used to analyze microstrip lines at millimeter wave frequencies. The dyadic Green's function accounts accurately for fringing fields and dielectric cover over the microstrip line. Using Rumsey's reaction concept an expression for the characteristic impedance is obtained. The numerical results are compared with the results reported earlier.					
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